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ON THE TIME-FREQUENCY DYNAMICS OF GROWTH CYCLES IN BRAZIL¹

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ARTICLE INFO	ABSTRACT
JEL classification: C02, O42, H60	We add to the empirical literature on growth, by addressing instrumentalized co-movements between growth versus fiscal, monetary, and credit variables in Brazil. We use partial wavelet coherency, phase-difference diagram, and regression coefficient to model growth cycles' reaction to isolated fluctuations in net debt to GDP, primary balance to GDP, inflation, SELIC, household, and enterprise credit to GDP over time and across frequencies. We provide a set of new stylized facts about the history of growth in Brazil over the period from 2004q1 and 2022q4, during which the country has gone through three different recessions. We find that in the periods of expansion, primary balance cycles lead in-phase growth cycles, while inflationary and household credit cycles lead out-of-phase growth cycles. During the recessions, we find a relevant positive role played by the enterprise credit, and a negative leadership of SELIC cycles on economic growth cycles.
Keywords:	
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1. Introduction

Researchers and policy makers are very concerned about the current worldwide macroeconomic scenario after a period of pandemic and war: widespread high inflation and high public debt, even in developed countries. For instance, according to Statistics News Release published by Organization for Economic Co-operation and Development (OECD), year-on-year inflation in the OECD as measured by the Consumer Price Index (CPI) reached 10.3% in November 2022. Concerning G7 economies, at the third quarter of 2022 Japan has maintained high gross central government debt to GDP, close to 218%, followed by UK, Italy, and U.S.: 154%, 143%, and 117%, respectively, according to the Quarterly Public Sector Debt Statistics (QPSD) database.

This dangerous combination is quite rare in developed economies, but that does not seem to be true for emerging markets. Empirically, according to Reinhart and Rogoff (2010), in emerging market countries, high public debt levels coincide with higher inflation, and they also call attention for external debt in emerging economies, due to the currency exposure. Theoretically, Araujo et al. (2022) claim that high public debt limiting low inflation is a common fact in emerging economies. To summarize, those fiscal and monetary numbers are worrisome because in most of the countries such values are high when compared to their respective historical series, and they are even more serious in the context of developing economies.

We are concerned about the worldwide situation, obviously, however, even more concerned about the macroeconomic situation in Brazil, the tenth largest economy in the world, according to the IMF.

During the period from 1980 to 2022, this economy has registered 10 recessions, according to the Economic Cycles Dating Committee (CODACE/FGV). While the longest crises lasted 11 quarters, the shortest ones lasted just one semester. Despite being very short (from 2020q1 to 2020q2), the last recession due to the COVID-19 pandemic caused the highest drop in the economic activity in Brazil: 10.7%. The recent fiscal scenario seems very worrying. Comparing the period before and after the fateful first semester of 2020, we see an increase in the net debt to GDP (from 44.98% to 51.71%), and in the quarter primary deficit to GDP (from 1.31% to 13.46%).

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On the other hand, official inflation reduced (from 1.77% to 1.24% per quarter), and the SELIC rate too (from 5.05% to 2.10% per year). However, this monetary side has rapidly deteriorated since then, given the supply shock due to lockdown succeeded by war. Inflation in the last quarter of 2022 was 1.63%, and in the year, the inflation of 5.78% was higher than the target ceiling, while the SELIC rate ended 2022 at 13.75% per year.

We need to understand how serious this combination of fiscal and monetary variables can be, and its possible consequences. We add to the empirical and theoretical literature on growth, transmission channel, and shock decomposition in Brazil, by looking at the dynamics of growth through the lens of a framework able to measure the complex instrumentalized co-movements between growth cycles versus fiscal, monetary and credit cycles, allowing for different lead-lag relationships in the time-frequency domain, from 2004-q1 to 2022-q4. To the best of our knowledge, our paper is the first to employ such an integrated analysis of this set of continuous wavelet tools to effectively estimate a dynamic multivariate growth reaction in the continuous time-frequency space for Brazilian economy.

Methodologically, first we use Global Wavelet Power Spectrum to study volatility distribution of the main variables. Second, we use multivariate coherency to measure the explanatory power of the suggested model. Finally, we use partial wavelet coherency, phase-difference diagram, and regression coefficient to model growth cycles' reaction to isolated fluctuations in net debt to GDP, primary balance to GDP, official inflation, SELIC interest rates, household, and enterprise credit to GDP over time and across frequencies.

Our main findings are very important to understand the different possible transmission mechanisms of supply or demand shocks in recessions, as well as the pass through in periods of expansion, mainly, in the current scenario in which the world economy is experiencing the results of an extremely rare combination of negative events whose consequences are not yet predictable nor measurable.

The paper proceeds as follows. In [Section 2](#), we present the related literature. In [Section 3](#), we describe the methodology. In [Section 4](#), we analyze the data. In [Section 5](#), we apply the wavelet tools to provide a continuous time-frequency assessment of the Brazilian growth cycles reaction. In [Section 6](#), we offer concluding remarks.

2. Literature

Theoretically, it is possible to identify transmission mechanisms involving most of the macroeconomic, fiscal, monetary and credit variables in the recent approach proposed in [Cochrane \(2021\)](#): the fiscal theory of price level. The kernel of such theory assumes that inflation adjusts so that the real value of all government debt, including money, equals the present value of current and future primary surpluses. This concept is quite old, and it is a rereading with a well-founded framework of a thought proposed by [Smith \(1776\)](#): “money is valuable because we need money to pay taxes”. In other words, eventually, all of the money outstanding today and all of the money promised by outstanding government debt must be soaked up by surpluses.

According to [Cochrane \(2021\)](#), fiscal theory does not necessarily predict a tight relationship between current debt or deficits and inflation. He argues that if the government runs a big deficit, but people trust that deficit will be repaid by higher subsequent surpluses, then people are happy to hold the extra debt rather than try to spend it, and there is no inflation. Governments that want to borrow, to raise revenue to fight wars or recessions, and do not want to create inflation, will credibly promise repayment. Moreover, such theory only predicts inflation when debt is larger than what people think the government will repay, i.e., if people become convinced that government will end up printing money to cover intractable deficits, they will see inflation in the future and so will try to get rid of dollars today, driving up the prices of goods, services, and eventually wages across the entire economy.

More specifically, as the government pays off maturing debt, the bondholders receive a lot of money. Normally, that money would be used to buy new debt. But if investors start to fear inflation, which can erode the returns on such bonds, they won't buy the new debt. Instead, they will try to buy other assets that are less sensitive to inflation. But there are only so many real assets around, and someone has to hold the stock of money and government debt. So, the prices of real assets will rise. Then, with “paper” wealth high, people will start spending more on goods and services. But there are only so many of those around, too, so the overall price level must rise. Thus, when short-term debt must be rolled over, fears of future inflation give us inflation today – and potentially quite a lot of inflation.

Regarding the role played by the interest rates, the nonlinear effect of debt on growth can be related to a response of market interest rates as economies reach debt tolerance limits, according to [Reinhart et al. \(2003\)](#). According to them, as countries reach debt tolerance limits, sharply rising interest rates, in turn, force painful fiscal adjustment in the form of tax hikes and spending cuts, or, in some cases, outright default. This pass-through is even more complex, since it may depend on the term structure of the interest rate, the maturity of such debt, and its solvency, i.e., its relationship with to the flow of surplus/deficits. Moreover, based on a Keynesian mindset, the Central Bank's mission is to control interest rates to provide just the right level of demand so that the economy does not grow too quickly and cause excessive inflation, and so that it does not grow too slowly and sink into recession. Regarding this issue, [Dizioli and Wang \(2023\)](#) estimate a DSGE model for Brazil, to compare how the adaptive learning mechanisms differ in relation to U.S., and what the implications are for the macroeconomic dynamics and monetary policy when a shock hits these economies.

The last piece in this puzzle is the credit market. On the one hand, [Bordo and Jeanne \(2002\)](#) propose a very interesting pass-through involving credit, inflation, and debt. They claim that higher credit availability boosts asset prices through liquidity, i.e., higher inflation, and the expectation of further rises in the prices of assets motivates raising debt by investors. However, during periods of falling asset prices (useful as collateral) one can expect expenditure cut back and borrowing reduction. Even more relevant to our analysis is understanding that the Central Bank manages monetary policy by changing benchmark interest rates, these rates affect long-term interest rates, and then mortgage, loan, and other rates faced by consumers and business borrowers. Lower interest rates drive higher "demand," and higher demand reduces "slack" in markets, and the pass-through depends on the credit market. Eventually these "tighter" markets put upward pressure on prices and wages, increasing inflation. Higher rates have the opposite effect.

Regarding the role of inflation in growth, according to recent fiscal theory explored in [Cochrane \(2022\)](#), the key reason serious inflation often accompanies serious economic difficulties is straightforward: Inflation is a form of sovereign default. Paying off bonds with currency that is worth half as much as it used to be is like defaulting on half of the debt. And sovereign default happens not in boom times but when economies and governments are in trouble. To summarize, this mechanism related to interest rates is also useful to understand the role played by inflation in GDP, which is a classic research agenda of monetary economics, as we can see in a very informative survey on this issue, [Lucas \(2000\)](#).

In this related empirical literature, [Reinhart and Rogoff \(2010\)](#) study the longer-term macroeconomic implications of much higher public debt, based on a very simple exercise using a multi-country historical dataset. They find that median growth rates for countries with public debt over roughly 90% of GDP are about one percent lower than otherwise. They also find no systematic relationship between high debt levels and inflation for 20 advanced economies (with individual exceptions, as the U.S.), but in emerging market countries, high public debt levels coincide with higher inflation. In line with this empirical exercise suggested by [Reinhart and Rogoff \(2010\)](#), [Matos et al. \(2022\)](#) propose a more technically refined analysis of such relationships in U.S., allowing greater freedom (over time and with different frequencies) to the co-movements of the cycles of: debt, inflation, GDP per capita, and growth. They can identify when and whether cycles of inflation and debt are leading or lagging growth and business cycles over the last 56 years in U.S.

Concerning the role of credit in growth, [Beck et al. \(2012\)](#) propose to address the role played by enterprise and household credit. They find that the role played by the enterprise credit to GDP is positive and higher than household credit growth elasticity, for a sample of 45 developed and developing countries. Complementing this related empirical literature, [Chen et al. \(2012\)](#) are one of the rare papers addressing frequency-varying co-movements between the credit market and macrofinance. They use a multivariate analysis accounting for the phase shift mechanism to identify causality between financial cycles and business cycles even with raw data at different frequencies. More recently, [Matos et al. \(2021\)](#) address instrumentalized co-movements across time and frequencies between macrofinance variables and household decisions in terms of consumer loans, home mortgage and its respective delinquency rates in U.S. They apply the same methodology used by us aiming to provide insights to stock market return prediction and asset pricing puzzles, and to detect new stylized facts about the last three decades of U.S. financial development and economic growth.

3. Methodology

3.1. Decomposing time series: the Fourier transform and the wavelet transform

The Fourier analysis is a powerful tool to model time series on frequency domain, based on the central idea that any periodic function can be expressed by an infinite sum of trigonometric functions. By defining a basis of sines and cosines of different frequencies, the Fourier transform capture the relative importance of each frequency on the original signals. Given a time series $x(t)$, the Continuous Fourier Transform is:

$$F_x(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-i\omega t} dt \quad (1)$$

where ω is the angular frequency and $e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t)$ by the Euler's formula. In opposite of the original signal, the Fourier transform summarizes the data as a function of frequency and does not preserve information in time. The function is reversible, which allow back-and-forth between the original and transformed signals, and it gives an effective localization in frequency. So, we can access the power spectra of the signal, which describe the power distribution on different frequency bands. Besides the appealing of Fourier Transform to evaluate financial time series on frequency domain, the function does not allow decompose the time series into different time scales, which limit his applicability to study signals that exhibit bursts of volatility, abrupt regime changes, or non-stationarity, etc (In and Kim, 2012). To reach a balance between time and frequency, the short-time Fourier transform (STFT) was developed to expand the transformation by frequency and time-shift, it slides a window across the time series and taking the Fourier transform of the windowed series The STFT is given by:

$$F_x^Y(\omega, \tau) = \int_{-\infty}^{+\infty} x(t) \gamma_{t,\omega}^*(t) dt \quad (2)$$

where $\gamma_{t,\omega}(t) = \gamma(t - \tau)e^{-i\omega t}$ with $\gamma(t)$ being an analysis window and *denotes the complex conjugate.

Although the STFT provides the time-localized frequency information for situations in which frequency components of a signal vary over time, the constant length window used by STFT results in a transform which is limited by the Heisenberg uncertainty principle. It means that is impossible both measures the exact frequency and exact time of occurrence of this frequency in a signal. So, the drawback of STFT is that is apply fixed length window in the signal processing, which results in a uniform partition of time-frequency space, have not been able to capture events when they happen to fall within the width of the window. To resolve that problem, the Wavelet transform has three additional features over Fourier transform: i) it can decompose the data into several time scales instead of the frequency domain (which allow us to investigate the behavior of a signal over various time scales); ii) it uses local base functions that adjust the window width to deal with different frequencies (this enables a more flexible approach to deal with high and low frequency components) and iii) it allows to work with non-stationary data. The latter feature is especially important to examine financial time series, once that heteroskedasticity, sudden regime shifts, structural breaks at unknown time points are common pattern trough the financial cycle paths.

Given a time series $x(t)$, the continuous wavelet transform (CWT) is defined as:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t) \psi_{\tau,s}^*(t) dt \quad (3)$$

where * denotes the complex conjugate, τ determines the position, s is the scaling factor and $\psi_{\tau,s}$ is the basis function suited to scale and shift the original signal, which allows the decomposition of the time series both in space and scale (Farge, 1992). To capture the high and low frequencies of the signal, it's utilized a mother wavelet that is stretched and shifted (Francis and Kim, 2012):

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t - \tau}{s}\right) \quad (4)$$

The factor $1/\sqrt{s}$ is added to guarantee preservation of the unit energy ($\|\psi_{\tau,s}\| = 1$). Low scales are captured rapidly changing detail generating a compressed wavelet ($|s| < 1$), capturing high frequencies movements, and

high scales capture slowly changing features ($|s| > 1$), or low frequencies movements (Rua, 2012). So, the CWT can be defined by:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} \psi\left(\frac{t - \tau}{s}\right) dt \quad (5)$$

The basis function $\psi_{\tau,s}$ must obey some criteria, such as:

- i) Admissibility: for an integrable function, it means that its average should be zero and the function need be localized in both time and frequency space.

$$C_\psi = \int_0^{+\infty} \frac{|H(\omega)|}{\omega} d\omega < \infty \quad (7)$$

where $H(\omega)$ is the Fourier transform of frequency ω . It's a necessary condition to satisfy the Dialect condition, which ensures that $\lim_{\omega \rightarrow 0} H(\omega) = 0$. Hence, the first condition of a wavelet function is $\int_{-\infty}^{+\infty} \psi(t) dt = 0$. If the energy of a function is defined as the squared function integrated over its domain, the second condition is that the wavelet function has unit energy ($\int_{-\infty}^{+\infty} |\psi(t)|^2 dt = 1$).

- ii) Similarity: the scale decomposition should be obtained by the translation and dilation of only one wavelet mother function. This dilation procedure allows an optimal compromise in view of the uncertainty principle: The wavelet transform gives very good spatial resolution in the small scales and very good scale resolution in the large scales (Farge, 1992).
- iii) Invertibility: Once that the energy of the original signal $x(t)$ is preserved by the wavelet transform if the admissibility condition is satisfied, we have $\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |W_x(\tau, s)|^2 \frac{d\tau ds}{s^2}$. Hence, there should be at least one reconstruction formula for recovering the signal exactly from its wavelet coefficients and for allowing the computation of energy or other invariants directly from them.
- iv) Regularity: The wavelet should be concentrated on some finite spatial domain and be sufficiently regular.
- v) Vanishing moments: The wavelet should have some vanishing high-order moments. This requirement allows the study of its high-order fluctuations and possible singularities in some high-order derivatives. It means that scale function is smoother.

There are many options of wavelet mother functions to select, which includes Daubechies, Haar, Mexican Hat, Morlet and Meyer wavelets. Aguiar-Conraria and Soares (2011) highlight the importance of that choice and suggest picked up an analytic wavelet to study the synchronism between oscillatory signals because its corresponding transform contains information on both amplitude and phase, providing an estimate of the instantaneous amplitude and instantaneous phase of the signal in the neighboring of each time/scale location (τ, s) . On subset of analytic wavelet, the Morlet wavelet mother is the most popular alternative because some properties, which is given by:

$$\psi_{\omega_0}(t) = \pi^{-1/4} e^{i\omega_0 t} e^{-t^2/2} \quad (8)$$

where the non-dimensional frequency ω_0 is set $\omega_0 = 6$ to satisfy the admissibility condition (Torrence and Compo, 1998). As the wavelet transform decomposes the original signal in a time-scale domain, which put us the necessity to convert scale into frequency. Lilly and Olhede (2009) points that this conversion can be made by associate the wavelet $\psi_{\tau,s}$ with one of three special frequencies (the peak frequency, the energy function, or the central instantaneous frequency), by using the formula $\omega(s) = \frac{\omega_\psi}{s}$, where ω_ψ denotes any of the three angular special frequency. By the usual "Fourier" frequency f (cycles per unit time) we have that $f(s) = \frac{\omega_\psi}{2\pi s}$. In this sense, the Morlet wavelet is an ideal alternative because it provides us a unique relation between frequency and scale (the peak frequency, the energy frequency and the central instantaneous frequency are all equal) which makes it easier the conversion from scales to frequencies. The choice of $\omega_0 = 6$ give us a conversion

ratio equal $f = \frac{6}{2\pi s} \approx \frac{1}{s}$, that direct correspondence between scale and frequency is ideal to simplify an effective interpretation of the results.

Finally, because the CWT is applied on finite-length time series, border distortions will occur due the fact that values of the transform at the beginning and the end of the sample are imprecisely computed, which involve artificial padding on the extremes of the sample (the most common is set zero to extend the time series). As larger scales decrease the amplitude near the edges as more zeroes enter the analysis (Torrence and Compo, 1998), the region that suffers from these edge effects is function of s . The Cone of Influence (COI) is the region of the wavelet spectrum in which edge effects become important by a factor of e^{-2} . In the case of the Morlet wavelet this is given by $\sqrt{2s}$.

3.2. Wavelet tools

Given our purpose, in our first exercise we propose analyzing individually the variance distribution of each economic, monetary, fiscal, and credit variable. In this context, the first wavelet measure useful is the wavelet power spectrum (WPS). It reports the variance distribution of the original time series $x(t)$ around the time-scale (or time-frequency) plane. Following Torrence and Compo (1998) we define the WPS by:

$$WPS_x(\tau, s) = |W_x(\tau, s)|^2 \quad (9)$$

where $W_x(\tau, s)$ is the continuous wavelet transform, τ determines the position, and s is the scaling factor. In sequence, aiming to compare the oscillation in energy among a range of bands (or frequency) we define the Global Wavelet Power Spectrum (GPWS), which takes the average of wavelet power spectrum over all times:

$$GPWS_x(\tau, s) = \int_{-\infty}^{+\infty} |W_x(\tau, s)|^2 d\tau \quad (10)$$

In our second empirical exercise, we start analyzing all variables of the model together. We aim to capture the interdependence and/or causality in a model with multiple (seven) time series, we follow Aguiar-Conraria and Soares (2014) and Aguiar-Conraria et al. (2018), by using high-order wavelet tools.

In other words, this multiple wavelet coherency allows us to investigate the dependency of one time series upon a set of other time series. Despite being a well-known tool, it is useful to compare the output of this technique to a type of R^2 in a regression estimation, i.e., a metric of the explanatory power of the suggested model, however capable of varying over time and in the range of frequencies.

To study the dependencies between two original time series $x(t)$ and $y(t)$ in time-scale/frequency plane, Torrence and Webster (1999) were the first to define the wavelet coherency. The measure that is associated to the cross-wavelet spectrum (XWT), which in turn can be derived by (Torrence and Compo, 1998):

$$W_{xy}(\tau, s) = W_x(\tau, s)W_y^*(\tau, s) \quad (11)$$

where $W_x(\cdot)$ and $W_y(\cdot)$ are continuous wavelet transform of $x(t)$ and $y(t)$, respectively, and $*$ denotes the conjugates complex. As the cross-wavelet transform is complex, we can express the XWT as $|W_{x,y}(\tau, s)|$. It computes the local covariance between two signals at each scale. The squared wavelet coherency is given by the squared of the wavelet cross-spectrum normalized by the individual power spectra. Following Torrence and Webster (1999) the squared wavelet coherency is denoted as:

$$R^2(\tau, s) = \frac{|S(s^{-1}W_{x,y}(\tau, s))|^2}{S(s^{-1}W_x(\tau, s)^2)S(s^{-1}W_y(\tau, s)^2)} \quad (12)$$

where $S(\cdot)$ expresses a smoothing operator in both time and scale, s^{-1} is a normalization factor ensuring the conversion to an energy density. Torrence and Webster (1999) notes that in numerator of the squared wavelet coherency, both the real and imaginary parts of the cross-wavelet transform are smoothed separately before taking the absolute value, while the smoothing operator is taking on square of the wavelet power spectra in denominator. By these definitions, it is ensured that $0 \leq R^2 \leq 1$. Aguiar-Conraria et al. (2018) define the extension of this concept to a model, in which we have to deal with three variables.

For instance, the squared multiple wavelet coherency between the series $y(t)$ and two other series $x(t)$ and $z(t)$, denoted by $R_{y(x,z)}^2$ is given by

$$R_{y(x,z)}^2 = \frac{R_{yx}^2 + R_{yz}^2 - 2\Re(\varrho_{yx}\varrho_{xz}\overline{\varrho_{yz}})}{1 - R_{xz}^2} \quad (13)$$

where ϱ_{yx} denotes the complex wavelet coherency of $y(t)$ and $x(t)$. The multiple wavelet coherency between $y(t)$, $x(t)$ and $z(t)$, denoted by $R_{y(x,z)}$ is the positive square root of the value reported in (13). The extension of this concept for a model with more than three variables it is straightforward.

In our third and most relevant exercise, we propose a conditional analysis based on the use of instruments or control variables, to isolate the individual effect of a fiscal or monetary or credit variable, for example. Although the wavelet coherency computes the degree of local linear correlation between two signals, it does not reveal patterns of lead-lag relationship neither if the movements are positives or negatives. We use the following phase-difference to examine the delays in the fluctuations between the two time-series:

$$\phi_{xy}(\tau, s) = \tan^{-1} \left(\frac{\Im \left\{ S \left(s^{-1} W_{x,y}(\tau, s) \right) \right\}}{\Re \left\{ S \left(s^{-1} W_{x,y}(\tau, s) \right) \right\}} \right) \quad (14)$$

The smoothed real (\Re) and imaginary (\Im) parts should already be calculated in the wavelet coherency function. Both $R^2(\tau, s)$ and $\phi_{xy}(\tau, s)$ are functions of the position index (τ) and scale (s). We also need the information on the signs of each part to completely determine the value of $\phi_{xy} \in [-\pi, \pi]$.

A phase-difference of zero indicates that the time-series move together at the specified frequency. If $\phi_{xy} \in (0, \frac{\pi}{2})$ the series move in phase, but the time-series y leads x , while if $\phi_{xy} \in (-\frac{\pi}{2}, 0)$ then it is x that is leading. A phase-difference of $\phi_{xy} = \pm\pi$ indicates an anti-phase relation. If $\phi_{xy} \in (\frac{\pi}{2}, \pi)$, then x is leading and time-series y is leading if $\phi_{xy} \in (-\pi, -\frac{\pi}{2})$.

Remember that we aim to capture the conditional or partial interdependence and/or causality between two time series in a model with multiple time series. A useful extension of this unconditional framework described is the partial wavelet coherency of x_1 and x_j allowing for all the other series, is denoted by:

$$R_{1j,q_j} = \frac{|\mathcal{L}_{j1}^d|}{\sqrt{\mathcal{L}_{11}^d} \sqrt{\mathcal{L}_{jj}^d}} \quad (15)$$

where d denotes the determinant of the matrix, and \mathcal{L}_{ij}^d denotes the cofactor of the element in position (i, j) . After controlling the influence of all the other time series ($x_i; i = 2, \dots, p$ and $i \neq j$), [Aguilar-Conraria et al. \(2018\)](#) define the *partial phase-difference* of x_1 over x_j as:

$$\phi_{1j,q_j} = \tan^{-1} \left(\frac{\Im(\varrho_{1j,q_j})}{\Re(\varrho_{1j,q_j})} \right) \quad (16)$$

The trigonometric interpretations of this partial framework (with a vector of instruments) are like those described for the unconditional model. Finally, following [Aguilar-Conraria et al. \(2018\)](#) we define the *partial wavelet gain* as follows:

$$G_{1j,q_j} = \frac{|\mathcal{L}_{j1}^d|}{\mathcal{L}_{11}^d} \quad (17)$$

This metric works like a kind of elasticity or partial derivative, however in the time-frequency domain.

4. Data

We use quarterly data for the Brazilian economy over the period 2004q1–2022q4 (76 observations). We work with data on the real GDP per capita, growth rate, net public debt to GDP, primary balance to GDP, inflation (IPCA), SELIC rate, household, and enterprise credit, both as a ratio to GDP.²

We report those time series in Fig. 1. According to the Economic Cycles Dating Committee (CODACE/FGV), there were 3 recessions in Brazil during this period from 2004 to 2022.³ A first recession between 2008q4 and 2009q1, due to the subprime crisis in 2007 in the United States with global repercussion. The second recession was the longest (from 2014q2 to 2016q4) and the most serious until then, since the 1980s, and its origin is given some fiscal and monetary policies in Brazil. More specifically, this long, and local crisis of the Brazilian economy is partially due to the measures taken to contain the previous subprime crisis, such as: the artificial fall in energy prices and interest rates, tax relief and the increase in government credit through public banks. Such responses allowed a quick recovery from the subprime recession, but generated fiscal problems and inflation, via the rapid easing of fixed administered prices after the crisis in U.S. The third and last recession, due to the worldwide COVID-19 pandemic was short; it happened during the 1st half of 2020, but it was the most serious of all. To summarize, the economic activity shrank by 5.5%, 8.6% and 10.7%, respectively, in these 3 recessions during these 19 years, according to CODACE.

We start the analysis of the time series reported in Fig. 1, by observing the recent fiscal scenario.

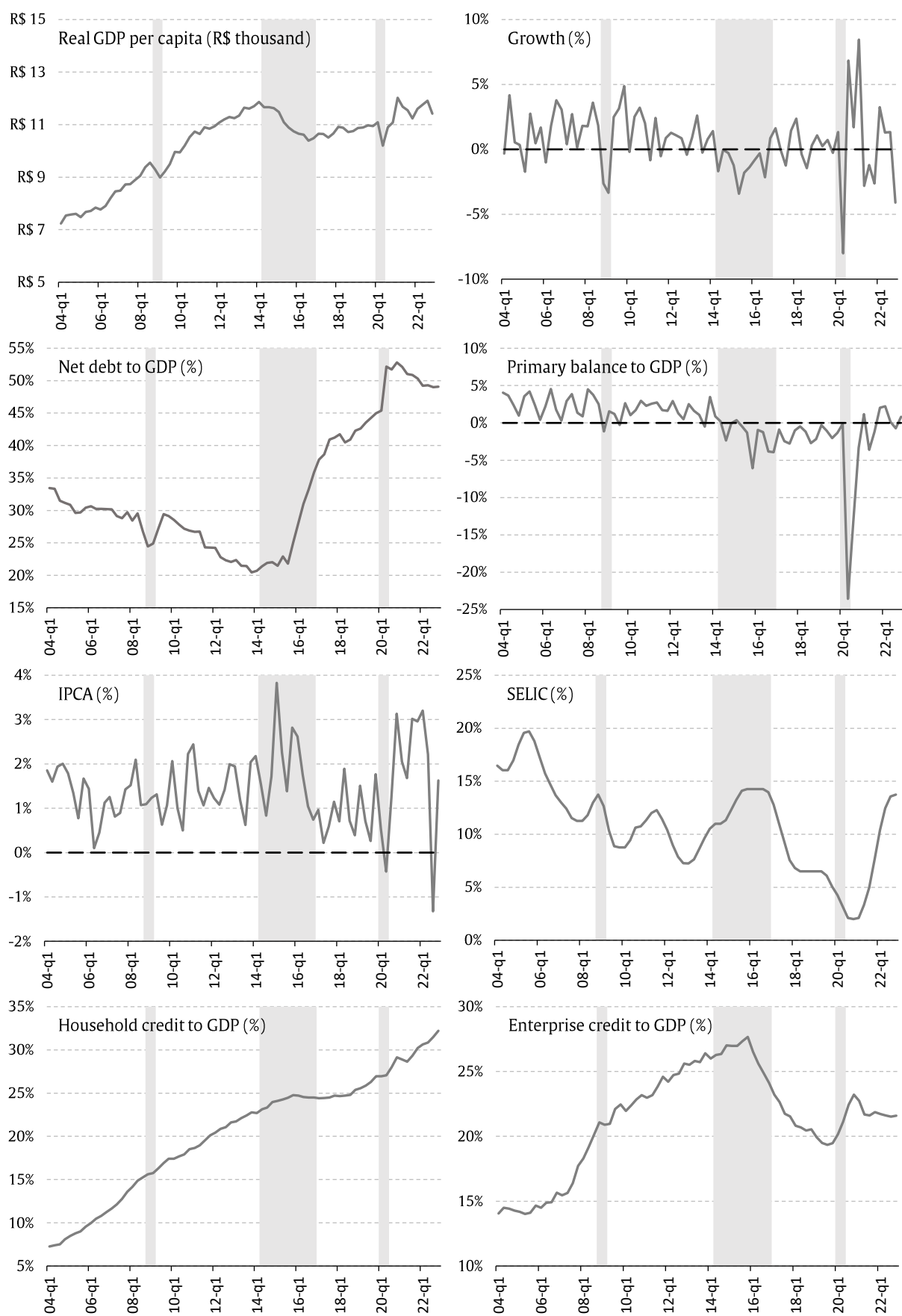
At the end of 2022, the net federal government debt was R\$ 4.83 trillion, equivalent to 49.08% of GDP (R\$ 9.84 trillion). This indebtedness as a ratio of GDP has been maintained since 2020 at a level close to 50%. This level is higher than the value reported until the pandemics (44.98% in the end of 2019). Moreover, it is a record if compared not only with the last few years, but with the entire historical series of almost 20 years. The behavior of debt immediately before and after each of the three recessions are similar, because in all cases, the debt increased, with emphasis on the increase associated with the second recession, ranging from 20.70% to 37.82% 12 quarters later (Table 1).

We complement the analysis of the fiscal scenario, observing the behavior of the primary result in the context of current fiscal rules. Brazil set a primary surplus target in 1999 – a key condition for repairing the damage to public accounts in the first years of the Real Plan. The healthy trajectory of public accounts was reinforced the following year with the Fiscal Responsibility Law that imposed some rules on the federal, state and municipal governments. Observing the time series used here, between 2004 and 2013, in only 3 quarters there was a negative primary result: 2008q4, 2009q3 and 2013q3. In 2013, the federal government again recorded a primary surplus, but this time it failed to reach the planned primary result target.

At the end of 2013, the National Congress approved a project that exempts the federal government from offsetting primary surplus targets that were not achieved by states, the Federal District and municipalities. In practice, the proposal reduces the federal government's primary surplus target from R\$155.8 billion to R\$108

² Quarterly real GDP per capita is measured from the monthly GDP series in current values (available from the Central Bank of Brazil – series N. 4380). We work with this variable in real values (2022-q4 local currency, R\$) using the monthly official inflation measure (IPCA). The quarterly real GDP is calculated by aggregating the respective monthly information. To calculate the quarterly GDP per capita, we use the quarterly population, obtained from the monthly series of population estimates available in the continuous PNAD of the Brazilian Institute of Geography and Statistics (IBGE). Finally, we use a standard procedure for this GDP series to deal with seasonality. Growth rate is measured by its variation. Concerning the debt, we use Public Sector Net Debt (% GDP) of the Federal Government (available from the Central Bank of Brazil – series N. 4504). This is a monthly series, and we use the last observation in each quarter. The quarterly nominal primary balance is calculated from the respective sum of the current monthly flow series of borrowing needs of the public sector (NFSP) without exchange devaluation (primary result) of the Federal Government (available from the Central Bank of Brazil – series N. 4640). The deficit as a ratio of GDP is calculated by dividing this series in real values (2022-q4 local currency, R\$) by the real quarterly GDP. Regarding the calculation of credit as a ratio of GDP, it is first necessary to aggregate the household (enterprise) credit series by region to obtain the monthly series of national credit. From then on, we apply the same procedure used to primary balance, already described. Inflation is given by the official measure, Broad Consumer Price Index (IPCA), also available from the Central Bank of Brazil. SELIC rate is given by the average of daily series of annual interest rate – SELIC target defined by COPOM (available from the Central Bank of Brazil – series N. 432).

³ The concept used by CODACE considers that there is a recession when a generalized drop in the level of economic activity is observed, regardless of having two consecutive quarters of negative GDP. Indicators such as consumption, investment, level of employment, civil construction performance, imports and exports, for example, are analyzed.



Notes: Shaded areas correspond to recession periods according to The Economic Cycles Dating Committee (CODACE/FGV).

Fig. 1. Real GDP per capita, growth, fiscal, monetary and credit variables (Source: Central Bank of Brazil and IBGE).

Billion, because the federal government will no longer be obliged to cover the portion of the economy of states and municipalities, which corresponds to the difference of R\$ 47.8 billion.

As of 2016, via Constitutional Amendment, the country began to adopt a new fiscal rule: the spending ceiling. The objective was to limit the growth of primary expenditure to the variation in inflation registered in the previous year. As a result of these new, less demanding fiscal rules, we began to see a succession of primary deficits. Only in 2021q1, after a record deficit of 23.57% in 2022q2 (the height of the pandemic), the country recorded primary surplus again. Nine years later, the country closed a year with a primary surplus. In 2022, there was a positive primary result of R\$ 54 billion.

Regarding the evolution of the primary deficit in the quarter before and after each recession, in the three crises, Brazilian government adopted a fiscal policy characterized by financing via debt increases and savings commitments, i.e., a reduction in the primary surplus.

Table 1. Behavior in the quarter before and after of the recessions in Brazil

	1st recession [2008q4 - 2009q1]		2nd recession [2014q2 - 2016q4]		3rd recession [2020q1 - 2020q2]	
	2008q3	2009q2	2014q1	2017q1	2019q4	2020q3
Real GDP per capita	R\$9.55	R\$9.21	R\$11.87	R\$10.65	R\$10.95	R\$10.90
Growth rate	1.85%	2.49%	1.40%	1.62%	-0.27%	6.84%
Net debt to GDP	26.78%	27.23%	20.70%	37.82%	44.98%	51.71%
Primary balance to GDP	2.52%	1.19%	0.89%	-0.88%	-1.31%	-13.46%
IPCA	1.07%	1.32%	2.18%	0.96%	1.77%	1.24%
SELIC	12.98%	10.36%	10.51%	12.79%	5.05%	2.10%
Household credit to GDP	15.25%	16.31%	22.68%	24.40%	26.95%	27.99%
Enterprise credit to GDP	20.14%	20.97%	26.00%	23.22%	19.48%	22.44%

Regarding to the monetary scenario, in [Fig.1](#) we observe the official inflation (IPCA) and the interest rate target (SELIC) defined by the Monetary Policy Committee (COPOM) of the Central Bank of Brazil, in view of the inflation target defined by the National Monetary Council (CMN). Based on the daily SELIC series, there are interest rates ranging between 15% and 20% in the first years of the sample. From the end of 2005, there is a downward trend until 2013, with cycles of occasional increases. After a new peak (14.25%) that persisted between July 1, 2015, and October 1, 2016, we see a new consistent downward trend. At its lowest level, this basic rate remained lower than 3% per year, during the period between June 1, 2020, and May 1, 2021. After that, there is a vertiginous escalation in the interest series, with successive and high variations, such as the 1.5% increase on October 1, 2021. The average of the daily series for the year 2022 indicates a SELIC of 12.54% per year, while the most current rate is of 13.75%. When we observe before and after each recession, except for the fiscal recession from 2014 to 2016, in the other crises this interest rate reduced.

Since 1999, when the Central Bank implemented the inflation targeting system in Brazil, there have been few times when the country's economy has managed to meet the target. According to IBGE, the 2022 IPCA once again breached the target ceiling (3.5% +/- 1.5%): the result was almost 5.8%. It was the seventh year, out of 24 of the target system, in which the tolerance limits were not respected. Of these seven, in six the overflow was upwards – 2001, 2002, 2003, 2015 and 2021, in addition to 2022. Only in 2017 the inflation (2.95%) was lower than the target (4.5% +/- 1.5%). Except for the small increase in inflation during the subprime recession, in the most recent crises, inflation has dropped ([Table 1](#)).

Regarding the economic activity, real GDP per capita has a robust and smooth growth, ranging from R\$ 7.2 thousand at the beginning of the sample to more than R\$ 9.5 thousand in mid-2008. Due to the subprime recession, there is first sharp drop in the series reported in [Fig. 1](#). After reaching the lowest level, close to R\$ 9 thousand in 2009-q1, GDP recovered again after the recession, until it fell again more sharply with the fiscal recession in the years 2014 to 2016. Comparing the quarter before and after this long recession, the cumulative drop in real GDP per capita was -10.42%. After a new slow but consistent recovery, there is a one-off drop of more than 8% in 2020-q2. The recent evolution between 2020-q3 and 2022-q4 has been characterized by the greatest oscillation ever measured in periods of stability/recovery. While in the other periods between

recessions, the standard deviation of quarterly growth used to range between 0.99% and 1.95%, in the post-COVID-19 period, this dispersion metric reached 4.13%. This excess of volatility is visible in Fig. 1.

Household credit as a ratio of GDP shows a smooth and constant growth behavior between 2004q1 (7.24%) and the beginning of 2016 (24.71%). This level stops growing and becomes more stable until 2018q3 (24.80%). Thereafter, we find a new growth trend, even stronger, until 2022q4 (32.21%). During all recessions, that credit always increased.

Enterprise credit was more than twice that of household credit at the beginning of 2004. This variable grows until the end of 2015, when it reaches its peak of 27.67% of GDP. In the last quarter of 2016, household credit surpassed enterprise credit for the first time. This pattern, unusual in most economies, continued until the end of 2022. In recessions, enterprise credit tends to grow, except for the 2014-2016 fiscal recession, when it fell from 26.00% in 2014q1 to 23.22% in 2017q1.

Finally, we follow the analysis promoted by Reinhart and Rogoff (2010), by grouping quarterly data on debt to GDP, primary deficit to GDP, SELIC, inflation, household and enterprise credit to GDP into four categories by quartile, depending on the growth rate (Table 2). In fiscal terms, the debt to GDP shows its highest average level in the lowest growth quartile, with some value stability in the other 3 quartiles, while the primary result to GDP shows increasing average values as growth varies positively. Under the monetary perspective, we find a pattern like that reported for the debt. Both basic interest rates and inflation show higher average values in the first quartile of growth, with lower and stable values in the other quartiles. It is important to note that this first quartile has 19 observations with a reduction in real GDP per capita, oscillating between -8.01% and -0.83%. Finally, both credit to families and companies show higher average values in the first quartile of growth, with a cyclical behavior in the other quartiles.

Observing the fiscal, monetary and credit behavior in this first quartile of growth reduction, the evidence suggests that the Brazilian government used the increase of deficit (tax reduction and/or increase in spending), the increase in debt, the inflation (as a kind of partial debt default from the point of view of the bondholders) and credit expansion as instruments of economic policy to soften the effects of the recessive cycle.

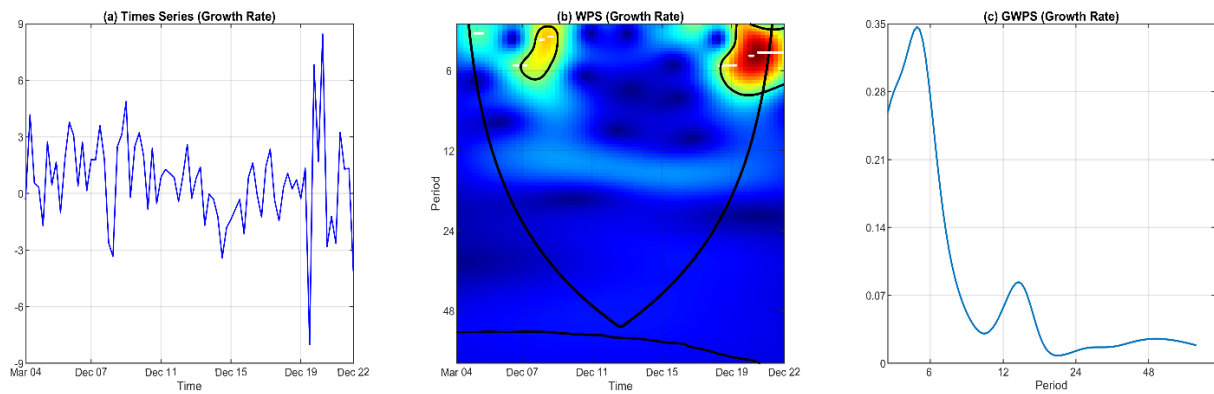
Table 2. Average values for each variable as the level of Brazilian growth rate varies (by quartile) over the period 2004-q1 to 2022-q4.

	-8.01% to -0.83%	-0.53% to 0.73%	0.78% to 1.83%	1.85% to 8.45%
Growth rate	-2.29%	0.00%	1.31%	3.48%
Net debt to GDP	34.05%	32.12%	33.15%	32.61%
Primary balance to GDP	-1.81%	0.27%	0.66%	1.26%
IPCA	1.77%	1.26%	1.31%	1.32%
SELIC	11.34%	11.00%	10.33%	10.58%
Household credit to GDP	22.90%	19.09%	21.84%	17.81%
Enterprise credit to GDP	22.65%	20.37%	21.95%	20.25%
Observations	19	19	19	19

5. Empirical Results

As we have explained in the methodology, our first empirical application aims to address the issue of time series volatility distribution. Fig. 2 shows WPS and GWPS for the quarterly growth series.

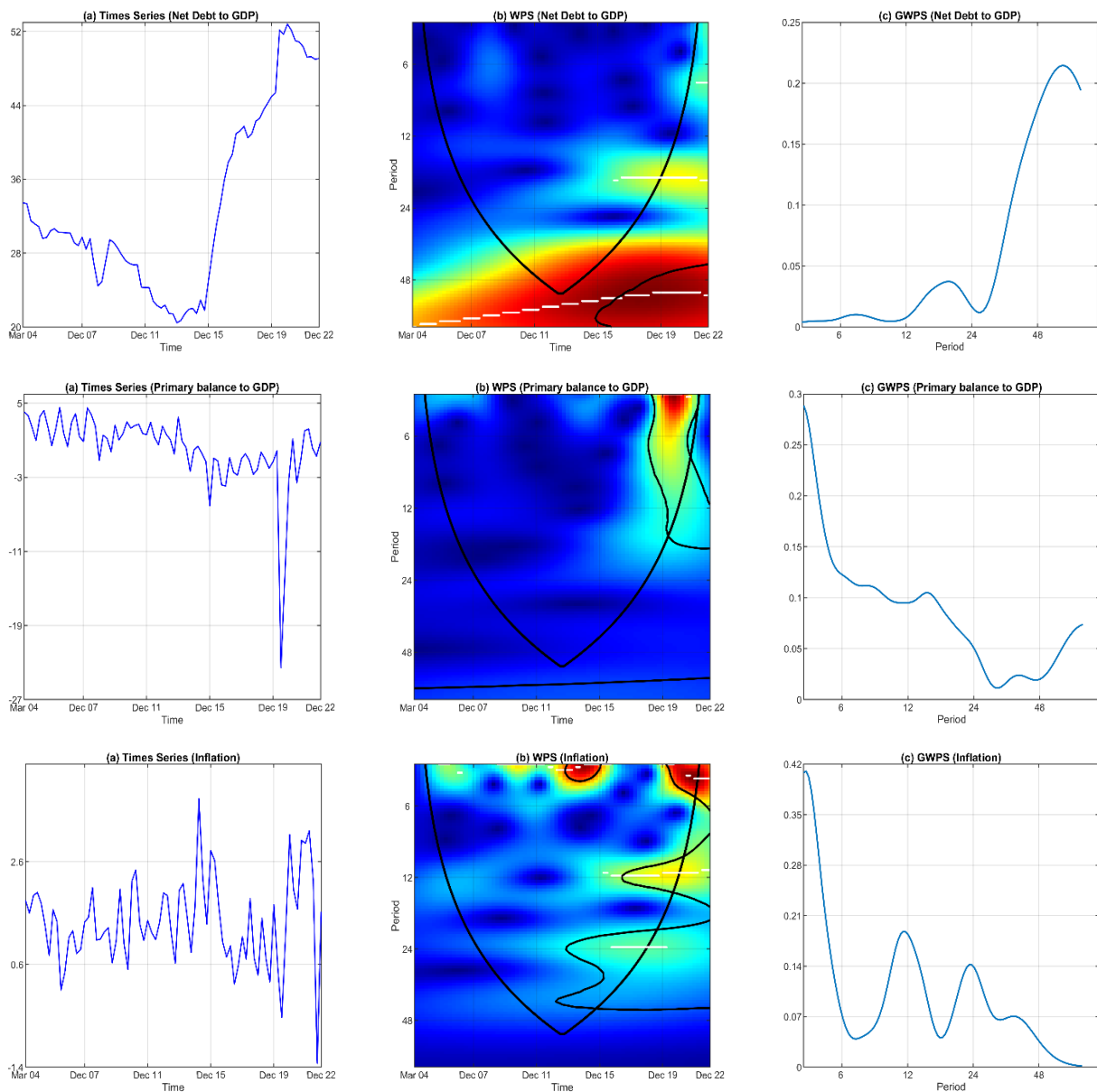
Remember that WPS reports the variance distribution of the original time series around the time-frequency plane, while GPWS takes the average of wavelet power spectrum over all times. We find that the spectrum varies along the time and frequency, indicating the non-stationarity of data and suitability of the method employed. The analysis of growth variance shows only two small areas with a significant peak in volatility during the subprime crisis period (late 2007 to mid-2009) and due to the recent pandemic, with volatility ranging from 20% and 35%, for a frequency corresponding to less than eight quarters, i.e., two years.



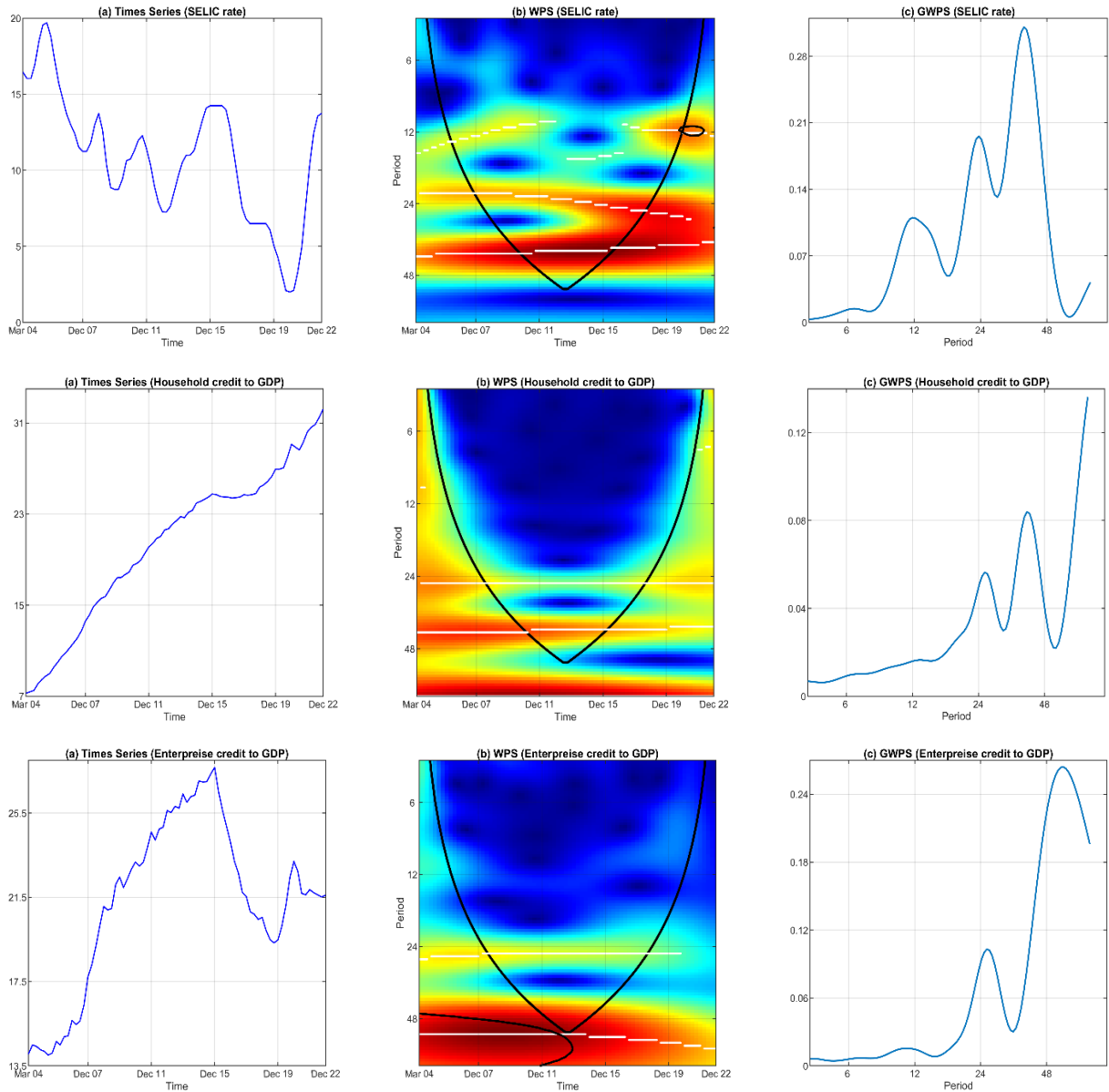
Notes: The black contours on the Wavelet Power Spectrum (WPS) plot refers to 5% significance and is theoretically obtained considering an AR (1) as the null hypotheses. In the heatmap, colder colors represent lower power while warmer colors represent higher power. The shaded area outside the Cone of Influence is subject to edge effects.

Fig. 2. Growth time series (a), respective WPS (b) and Global WPS (c).

Fig. 3 shows WPS and GWPS for all fiscal and monetary variable used here.



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Notes: The black contours on the Wavelet Power Spectrum (WPS) plot refers to 5% significance and is theoretically obtained considering an AR (1) as the null hypotheses. In the heatmap, colder colors represent lower power while warmer colors represent higher power. The shaded area outside the Cone of Influence is subject to edge effects.

Fig. 3. Fiscal, monetary and credit variables time series (a), respective WPS (b) and Global WPS (c).

Although the color map shows a region with a very low frequency (red area) in the middle of the period, there is no statistical significance in the indebtedness WPS, considering the region determined by the cone of influence. Regarding the primary balance, over those almost 20 years, only during the pandemic period it we identify a greater oscillation (statistically significant). We find greater volatility (ranging between 10% and 30%) in the area with high frequency (lower than 12 quarters, or three years). The volatility analysis of the monetary variables suggests curious patterns. In the case of inflation, there is greater intensity in a very specific moment in 2014 and 2015 (fiscal crisis) and a last stronger in 2020 and 2021, with a frequency of up to 6 quarters in both moments. At the height of volatility, we observe a value of up to 40% in higher frequency. Concerning the basic interest rate in Brazil, despite the extensive red area from 2007 to 2015, there is no significance. For both credit variables, there is no area with a significant volatility peak at 5% within the cone of influence region.

In our second empirical exercise, we address the issue of the joint explanatory power of the fiscal, credit and monetary variables used. This feature resembles and is comparable to the explanatory power of temporal regressions, measured by R^2 , for instance.

In our case, Fig. 4 shows the results on multivariate coherency, which ranges from low (blue) to high (red) values and the respective cone of influence is shown with a black line, designating the 5% significance level. The color and the significance in the heat map measure the degree of adjustment in the time-frequency domain of the debt, deficit, inflation, interest rate and growth rate. After having observed several maps like this, applied to the American economy, it is common we see many areas of blue color and only some areas of red color. However, in this map, there are only punctual areas of blue color. Therefore, there is a potential adjustment of the model involving these variables. There are some regions with significant coherency at 5%. We highlight a region with a frequency of up to 6 quarters in the period of the subprime crisis, another region with a frequency greater than 12 quarters during the fiscal crisis, as well as a punctual area with high frequency during the pandemic. There is still a large significant region with low frequency, above 24 quarters.

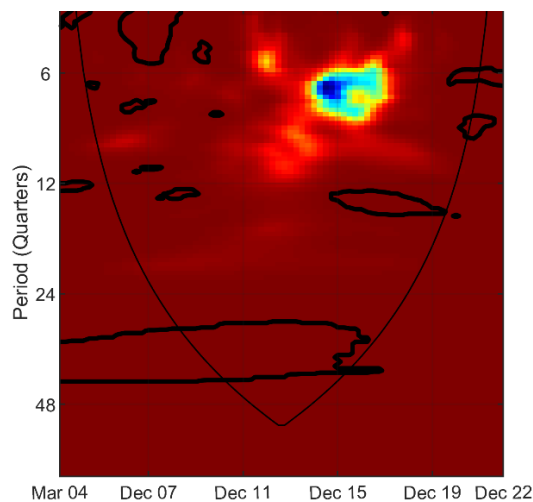


Fig. 4. Multiple coherency.

To summarize, the presence of successive areas (Fig. 4) with significance statistics in the multiple coherency denotes a good overall fit in the model and the next step is analyzing partial coherency, partial phase-difference with two standard deviations and partial gain (Fig. 5, Fig. 6, and Fig. 7). The coherency range from low (blue) to high (red) values and the respective cone of influence is shown with a black line, designating the 5% significance level. The purpose is modelling the linkage between growth cycles and the cycle of each one of the fiscal and monetary variables, controlling for the influence of other variables.

More specifically, in Fig. 5 (top), we model the co-movement between growth and debt cycles, controlled by primary balance, SELIC, inflation, household and enterprise credit cycles, and in Fig. 5 (bottom), we model the co-movement between growth and primary balance cycles, controlled by debt, SELIC, inflation, household and enterprise credit cycles. In an analogous way, in Fig. 6 (top), we model the co-movement between growth and inflation cycles, controlled by debt, primary balance, SELIC, household and enterprise credit cycles, while in Fig. 6 (bottom), we model the co-movement between growth and SELIC cycles, controlled by debt, primary balance, inflation, household and enterprise credit cycles. Finally, in Fig. 7 (top), we model the co-movement between growth and household credit cycles, controlled by debt, primary balance, SELIC, inflation, and enterprise credit cycles, while in Fig. 7 (bottom), we model the co-movement between growth and enterprise credit cycles, controlled by debt, primary balance, inflation, SELIC, and household credit cycles.

The way to understand each result is similar, and simple. First, we need to look at the heat map (left side). For each red area, we identify the respective period and frequency. For a frequency: i) lower than 12 quarters, the next step is looking at the first graph of partial phase (top graphic), ii) between 12 and 24 quarters the next step is looking at the second graph of partial phase (intermediate graphic), and iii) higher than 24 quarters, the next step is looking at the third and last graph of partial phase (bottom graphic). This same procedure goes for the graphs of partial gain reported (right side), where we can see the intensity of the elasticity.

As we have already said, in the first region $(\frac{\pi}{2}, \pi)$ we have an anti-phase or negative relation with x leading y . In the second region $(0, \frac{\pi}{2})$ both series move in phase, with y leading x . In the third region $(-\frac{\pi}{2}, 0)$ x is leading y in the same direction. And finally, in the last region $(-\pi, -\frac{\pi}{2})$ we have negative relation with y leading x .

As an example, about growth and enterprise credit, observing Fig. 7 (bottom), we identify a very small and punctual significative area (red color) from 2005q2 to 2009q2 with a frequency between 6 and 9 quarters. Looking at the first phase-difference graph in this same period, we can conclude, based on the line on the third region $(-\frac{\pi}{2}, 0)$, that we have a phase or positive relation with x (enterprise credit) leading y (growth). Observing the correct graph of partial gain (first one), we see an elasticity ranging between 1.5 and 4.2. In other words, our findings suggest that enterprise credit cycles in Brazil are able to lead or anticipate in the same direction the growth cycles from 2005q2 to 2009q2, controlled by SELIC, inflation, debt, primary balance, and household credit. In Table 2, we summarize our main conclusions based on the wavelet framework.

Our analysis follows a chronological rule, trying to isolate the role of each variable in the expansion or recession environment. Since the literature on the transmission channel seems to observe short- and mid-term relationships, and given that at the frequency above 16 quarters, we do not identify such interesting evidence, here, we pay more attention to co-movements with frequency up to four years. Our purpose is following the historical shock decomposition literature, shedding light about the different timing of fiscal, monetary and credit policy responses in Brazil in the 19-year period, from 2004 to 2022, and in this sense the partial gain is useful to measure the intensity of the conditional lead-lag relationships.⁴

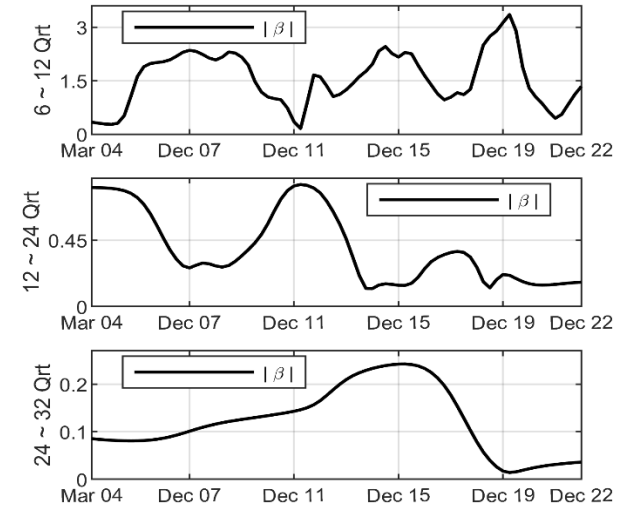
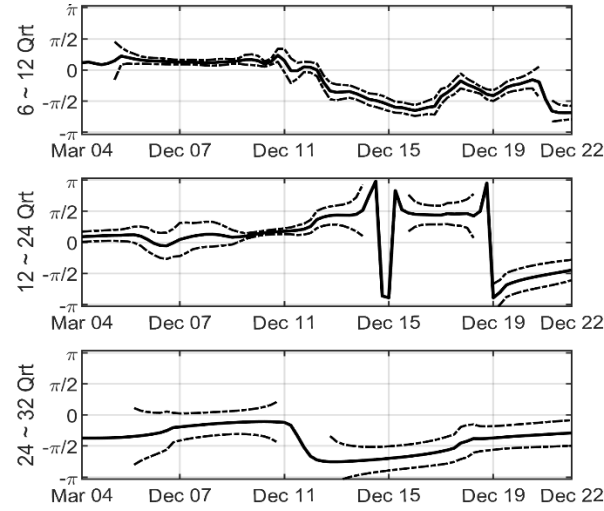
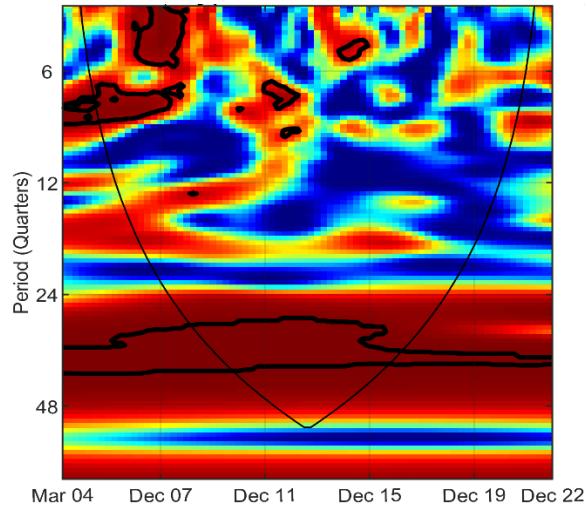
In this initial period of expansion (2004q1 – 2008q3), we identify that the economic growth cycles lead in-phase the debt cycles from the end of 2005 and the SELIC cycles from mid-2006. In both cases these co-movements remain up to the beginning of the subprime recession. Even during this expansion, from 2005 onwards, the primary result and enterprise credit cycles can lead in-phase the growth cycles, while household credit cycles lead out-of-phase growth cycles. From 2007 and in the beginning of 2008, we still find inflationary cycles leading out-of-phase growth cycles. It is interesting, because the subprime recession has been identified before in the US economy (the end of 2007) and despite the respective strong demand shock, given the generalized job loss and bank runs for instance, quarterly inflation in Brazil ranged between 0.81% (2007q2) and 2.09% (2008q2). To summarize the context that precedes the recession in 2008q4, although the SELIC interest rate having risen from 11.25% in mid-April 2008 to 13.75% in less than five months later, the increase of the inflation and the household credit seem to be able to minimize the positive effects of the increase in enterprise credit and fiscal austerity, with primary surpluses.

After this first expansion, and the short worldwide recession due to subprime in the U.S. from 2008q4, there is a new expansion between 2009q2 and 2014q2. As we have found during the previous expansion period, again we identify that the economic growth cycles can lead in-phase the SELIC cycles from mid-2012 to mid-2013. Moreover, we find that inflationary cycles can lead out-of-phase growth cycles from 2010 to mid-2013, minimizing the in-phase leadership of primary balances from 2011. We also find punctual evidence on the negative leadership of enterprise credit during the first half of 2011.

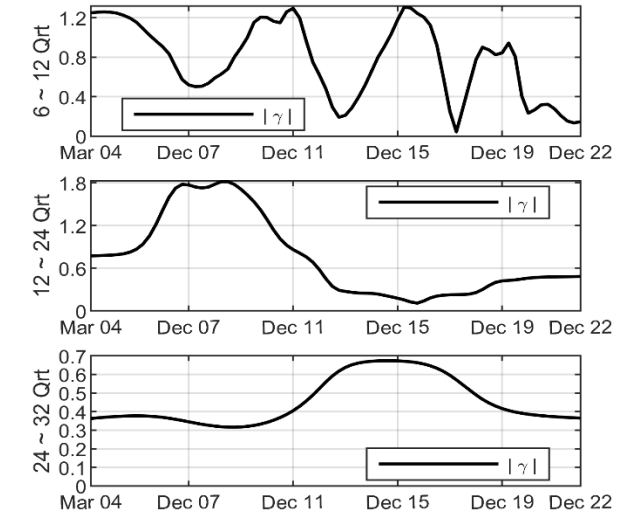
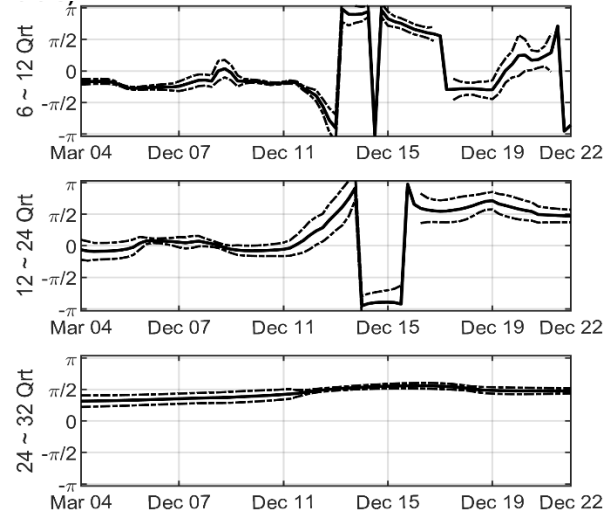
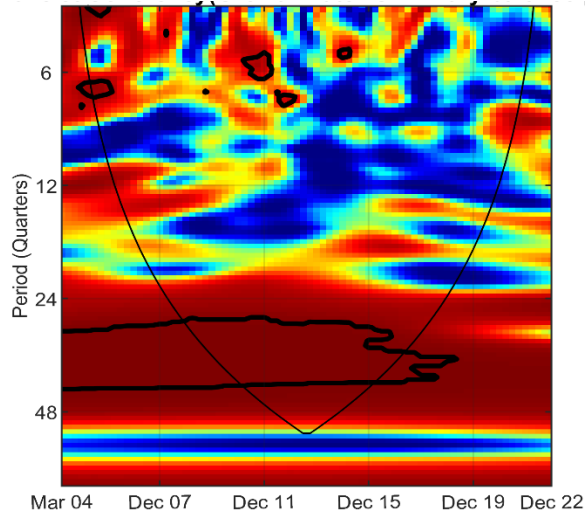
There is a second long and complex local recession, due to misguided fiscal and monetary policies, which extends from 2014q3 to 2016 q4. During this worrying period, we remain finding that enterprise credit cycles can lead in-phase the growth cycles, while SELIC cycles lead out-of-phase economic cycles. We went through a new expansion that lasted from 2017q1 to 2019q4, when a new short and deep recession emerged in the first half of 2020, characterized by a supply and demand shock. During the period between 2020q3 and 2002q4, there is a new economic expansion in Brazil, and during this las period, we find that inflationary cycles can lead in-phase growth cycles from 2020q4 to 2021q3. This evidence should not be seen as inflation as a growth driver, but as evidence that after the record drop-in economic activity and after the lockdown policy, and the consequent supply shock, prices reacted by increasing before the economic recovery.

In Fig. 8, we summarize this analysis.

⁴ Regarding the partial gains, if we were to estimate a regression of a growth model, the values of the conditional partial gains could be interpreted as parameters or partial derivatives that vary in the time and frequency dimensions.

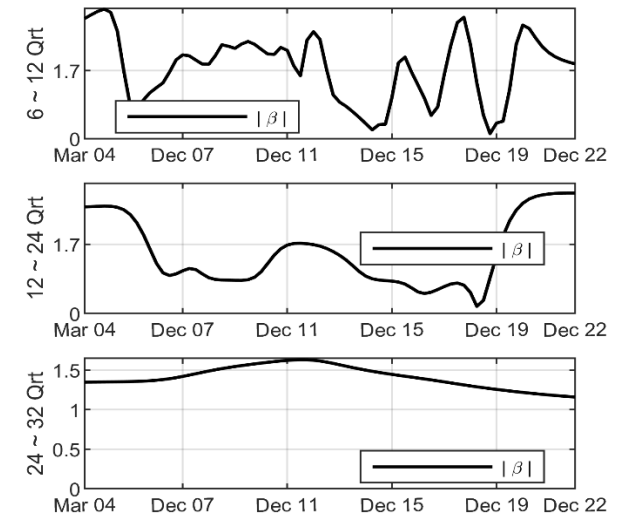
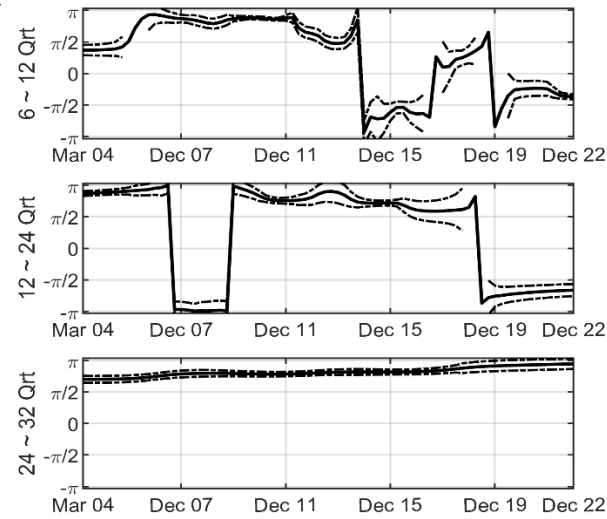
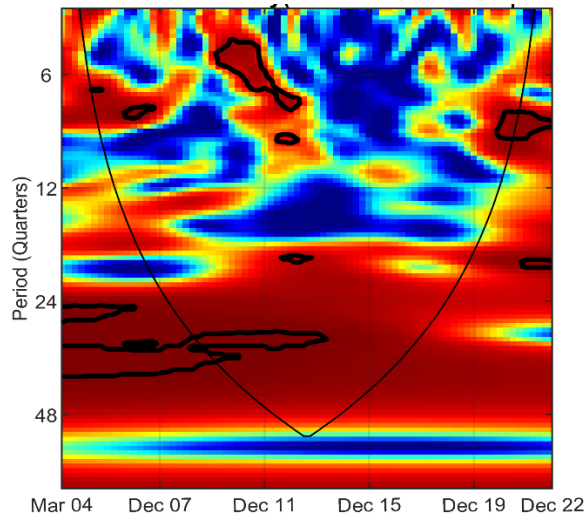


5.a. Growth rate (y) vs net debt to GDP (x) | Controls

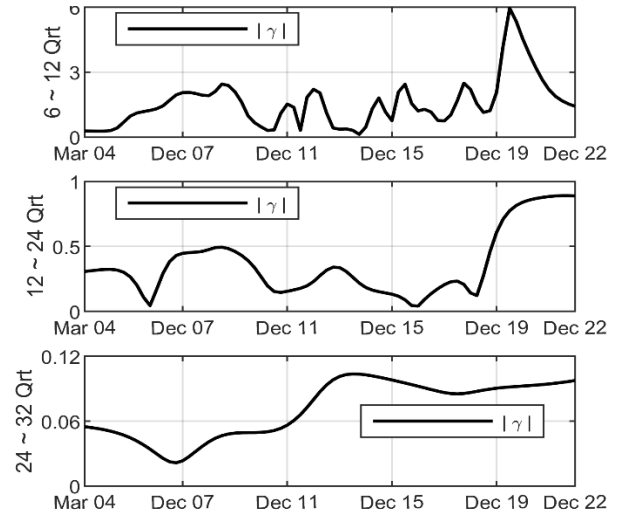
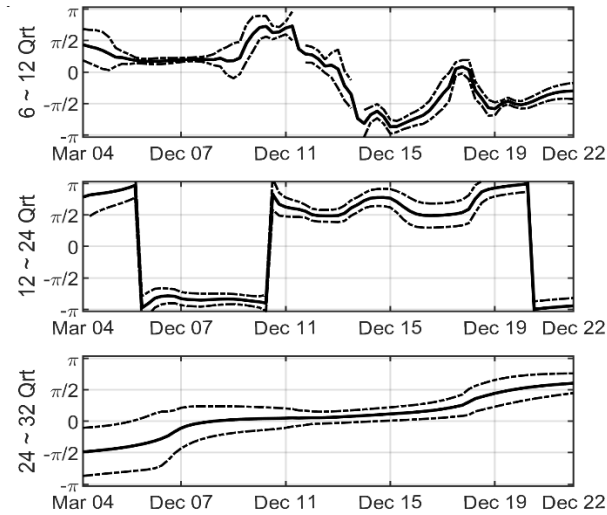
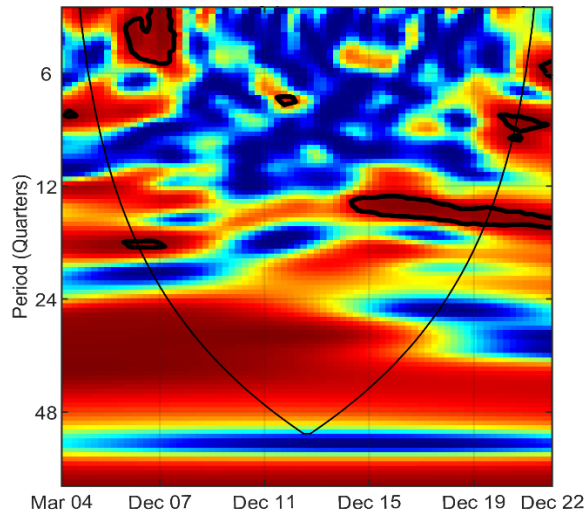


5.b. Growth rate (y) vs primary balance to GDP (x) | Controls

Fig. 5. Growth cycles vs fiscal cycles. Wavelet coherence (left side), Phase-difference (center) and Gain (right side).

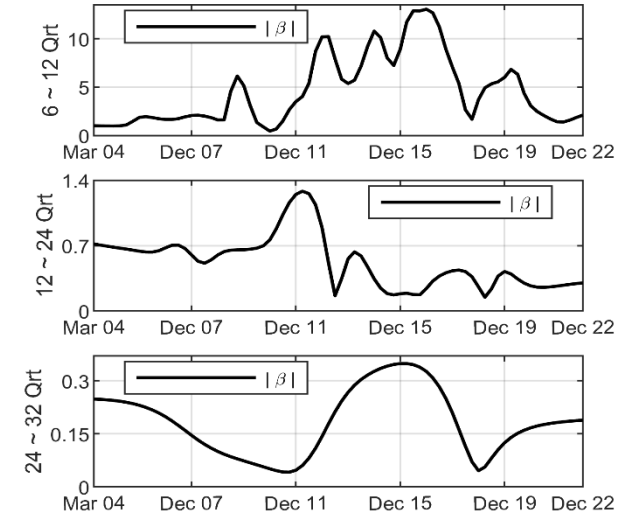
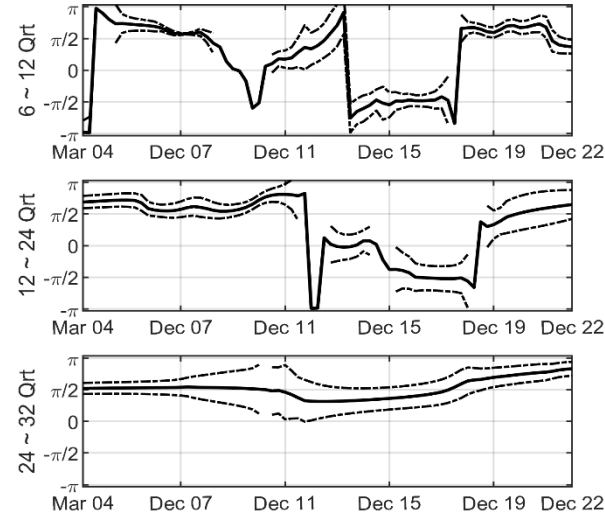
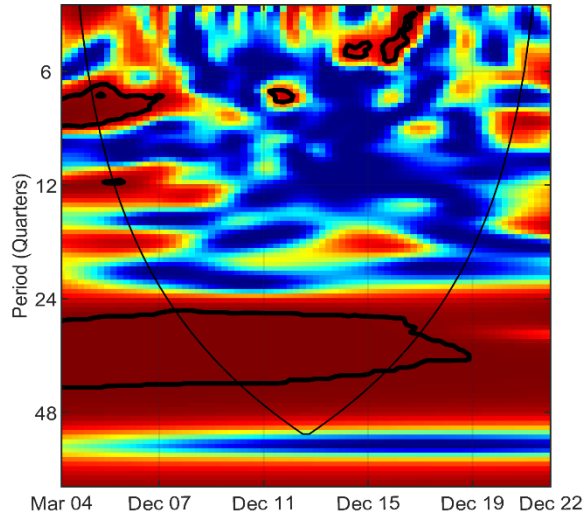


6.a. Growth rate (γ) vs inflation (x) | Controls

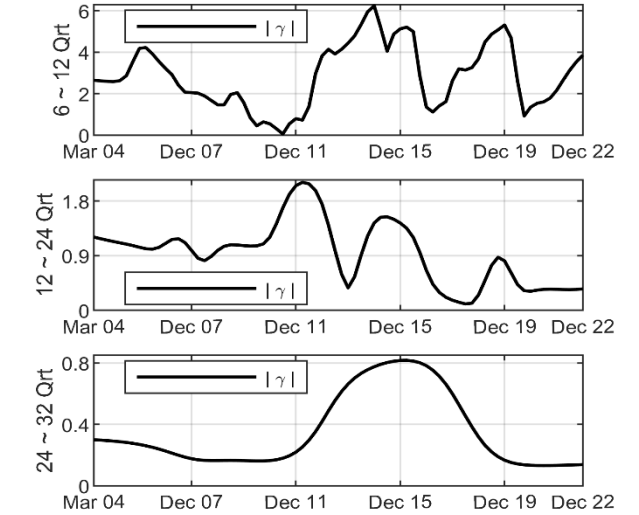
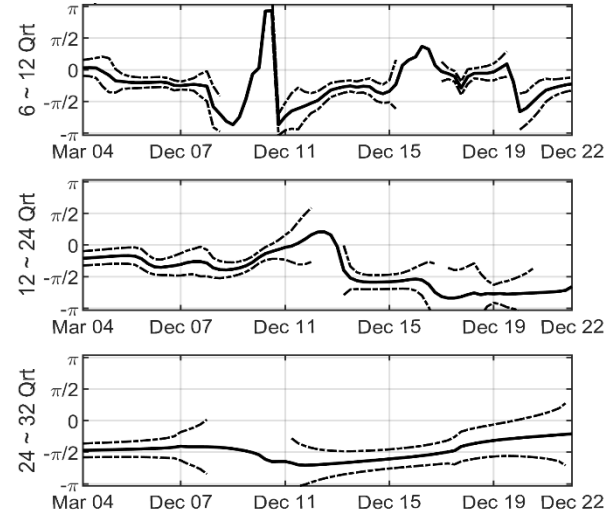
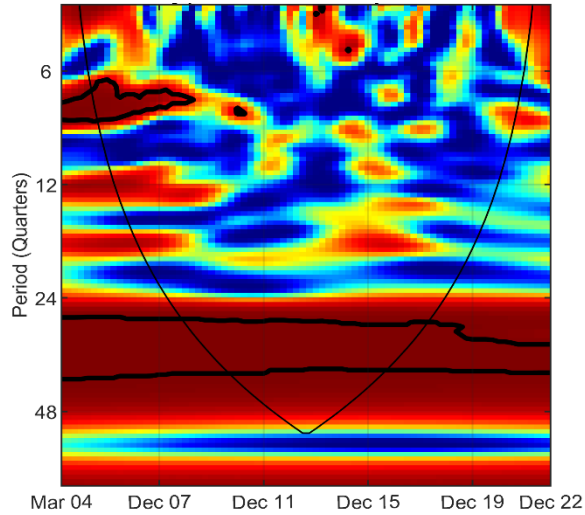


6.b. Growth rate (γ) vs SELIC rate (x) | Controls

Fig. 6. Growth cycles vs monetary cycles. Wavelet coherence (left side), Phase-difference (center) and Gain (right side).



7.a. Growth rate (y) vs household credit to GDP (x) | Controls

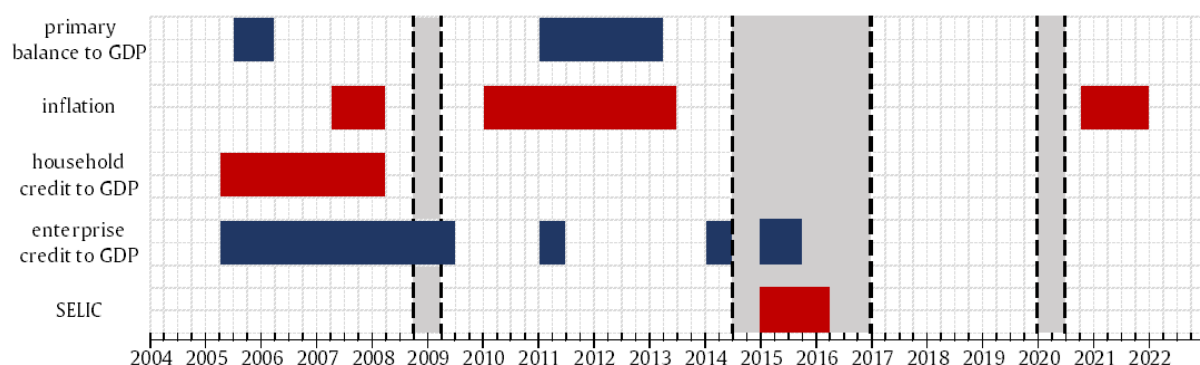


7.b. Growth rate (y) vs enterprise credit to GDP (x) | Controls

Fig. 7. Growth cycles vs credit cycles. Wavelet coherence (left side), Phase-difference (center) and Gain (right side).

Table 2. Main results for wavelet analysis (summary of Fig. 5, Fig. 6 and Fig. 7).

Period	Frequency	Co-movement	Elasticity
Growth cycles (y) versus debt cycles (x) - Fig. 5 (top)			
2005q3 - 2008q4	0 - 9 (quarters)	Growth cycles lead in-phase debt cycles	2.0 - 2.5
2011q4 - 2013q2	6 - 10 (quarters)	Phasic co-movement without leadership	-
2014q4 - 2015q4	3 - 6 (quarters)	Inconclusive	-
2009q3 - 2011q3	30 - 40 (quarters)	Phasic co-movement without leadership	-
2011q4 - 2016q2	30 - 40 (quarters)	Inconclusive	-
Growth cycles (y) versus primary balance cycles (x) - Fig. 5 (bottom)			
2005q2 - 2006q1	6 - 9 (quarters)	Primary balance cycles lead in-phase growth cycles	1.0 - 1.3
2011q1 - 2013q1	4 - 9 (quarters)	Primary balance cycles lead in-phase growth cycles	0.7 - 1.4
2014q4 - 2015q2	3 - 6 (quarters)	Antiphase co-movement without leadership	-
2008q1 - 2010q1	28 - 40 (quarters)	Growth cycles lead in-phase primary balance cycles	0.3 - 0.4
2010q2 - 2018q2	28 - 40 (quarters)	Inconclusive	-
Growth cycles (y) versus inflation cycles (x) - Fig. 6 (top)			
2006q2 - 2007q1	8 - 9 (quarters)	Antiphase co-movement without leadership	-
2007q2 - 2007q4	8 - 9 (quarters)	Inflation cycles lead out-of-phase growth cycles	1.7 - 2.0
2010q1 - 2013q2	3 - 9 (quarters)	Inflation cycles lead out-of-phase growth cycles	1.6 - 2.4
2012q3 - 2012q4	18 - 20 (quarters)	Inflation cycles lead out-of-phase growth cycles	close to 1.7
2013q1 - 2013q4	18 - 20 (quarters)	Antiphase co-movement without leadership	-
2009q2 - 2014q1	28 - 36 (quarters)	Inflation cycles lead out-of-phase growth cycles	1.3 - 1.7
2020q4 - 2021q3	8 - 10 (quarters)	Inflation cycles lead in-phase growth cycles	1.7 - 2.5
Growth cycles (y) versus SELIC cycles (x) - Fig. 6 (bottom)			
2006q2 - 2008q3	0 - 6 (quarters)	Growth cycles lead in-phase SELIC cycles	1.0 - 2.3
2012q3 - 2013q2	8 - 9 (quarters)	Growth cycles lead in-phase SELIC cycles	0.5 - 2.0
2007q1 - 2008q2	18 - 20 (quarters)	Growth cycles lead out-of-phase SELIC cycles	0.3 - 0.5
2015q1 - 2016q1	12 - 16 (quarters)	SELIC cycles lead out-of-phase growth cycles	0.1 - 0.2
2016q2 - 2019q1	12 - 16 (quarters)	Inconclusive	-
2019q2 - 2020q3	12 - 16 (quarters)	Antiphase co-movement without leadership	-
2020q4 - 2021q3	8 - 10 (quarters)	Inconclusive	-
Growth cycles (y) versus household credit cycles (x) - Fig. 7 (top)			
2005q2 - 2008q1	6 - 12 (quarters)	Household credit cycles lead out-of-phase growth cycles	close to 2.0
2012q1 - 2013q1	6 - 8 (quarters)	Antiphase co-movement without leadership	-
2015q1 - 2017q4	0 - 6 (quarters)	Inconclusive	-
2008q2 - 2017q4	24 - 40 (quarters)	Inconclusive	-
Growth cycles (y) versus enterprise credit cycles (x) - Fig. 7 (bottom)			
2005q2 - 2009q2	6 - 9 (quarters)	Enterprise credit cycles lead in-phase growth cycles	1.5 - 4.2
2011q1 - 2011q2	7 - 8 (quarters)	Enterprise credit cycles lead out-of-phase growth cycles	0 - 0.5
2014q1 - 2014q2	0 - 2 (quarters)	Enterprise credit cycles lead in-phase growth cycles	4.5 - 5.5
2015q1 - 2015q2	4 - 5 (quarters)	Enterprise credit cycles lead in-phase growth cycles	4.0 - 6.0
2008q4 - 2018q2	26 - 40 (quarters)	Inconclusive	-



Notes: Gray areas correspond to recession periods according to The Economic Cycles Dating Committee (CODACE/FGV). Blue areas mean in-phase and red areas mean out-of-phase leadership.

Fig. 8. Summary of the variables with short-term leadership on growth cycles

6. Conclusion

The current worldwide macroeconomic scenario after the pandemic and the war seems worrying given the worldwide high inflation and public debt, a combination which is more common, and it could be more severe in less developed economies, according to the related empirical and theoretical literature.

In Brazil, this scenario is very complex, based on the behavior of growth, debt, nominal deficit, SELIC rate and inflation during the period before and after the last recession due to the pandemic, and looking at the future, the expectations are not so optimistic. Regarding economic activity, according to the Focus forecast report by the Central Bank, dated February 03, 2023, the expected real GDP growth trajectory from 2023 to 2025 is: 0.79%, 1.50% and 1.89%, respectively. The expectation for the IPCA is 5.78% for the end of 2023, reducing to 3.93% and 3.50% in the years 2024 and 2025, respectively. The same report reports a gradual reduction in interest rates, with a forecast of 12.50% at the end of 2023, rising to 9.75% in 2024 and closing 2025 at 9.00%. Fiscal forecasts are even less optimistic. The public sector net debt should continue to grow, with an expectation of 61.45% in 2023, 64.38% in 2024 and 66.60% in 2025, while the nominal result forecasts a deficit over the next 3 years, with an apparent reduction: -8.10% (2023), -7.05% (2024) and -6.00% (2025).

In this context, we believe it is relevant studying the co-movements between growth cycles versus fiscal and monetary cycles. We find that rarely excessive indebtedness, high nominal deficit, high interest rates, and high inflation can lead positively boost the Brazilian economy. The evidence reported here for a period of almost 20 years, marked by three economic crises, suggests that the current fiscal and monetary scenario seems to be more likely to have a negative influence on growth cycles. The best way to mitigate the macroeconomic consequences due to high indebtedness, high inflation - a sign of partial debt default with bondholders -, unstable and negative flows of nominal results and extremely high interest rates is credibility and harmony in the conduct of fiscal and monetary policies. Only based on credibility, it is possible trying a pass-through between these variables with lower inflationary pressure and less need for indebtedness and/or stronger fiscal measures involving spending and revenue. Otherwise, without credibility, fiscal and monetary costs are very high, and negative effects on GDP growth must seem inevitable in Brazil. Secondly, with bondholders motivated to reduce their positions in government bonds, it is important to have complete and broad (financial) markets, which can better accommodate the increased demand for financial assets. This subsequently reduces, or at least delays, the increase in demand for goods and services, which would lead to inflation.

Finally, this research agenda suggested here can be extended from disaggregation of the variables. For instance, the primary result can be analyzed from the perspective of individual primary revenues and expenditures. Regarding the federal government's public debt, making use of internal and external debt data, as well as disaggregating by maturity (short-term, mid-term and long-term) can be very informative. Regarding the monetary variables, the use of the forward structure of the interest rate may be essential, in view of its predictive capacity of recession in some economies, as well as on the expectations of bondholders. Official inflation can still be decomposed considering that there are managed prices, for example. Regarding the credit market, both household and enterprise loans can be used, considering earmarked and non-earmarked values.

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We add to the empirical literature on growth, by addressing instrumentalized co-movements between growth versus fiscal, monetary, and credit variables in Brazil. We use partial wavelet coherency, phase-difference diagram, and regression coefficient to model growth cycles' reaction to isolated fluctuations in net debt to GDP, primary balance to GDP, inflation, SELIC, household, and enterprise credit to GDP over time and across frequencies. We provide a set of new stylized facts about the history of growth in Brazil over the period from 2004q1 and 2022q4, during which the country has gone through three different recessions. We find that in the periods of expansion, primary balance cycles lead in-phase growth cycles, while inflationary and household credit cycles lead out-of-phase growth cycles. During the recessions, we find a relevant positive role played by enterprise credit, and a negative leadership of SELIC cycles on economic growth cycles.