UNIVERSIDADE FEDERAL DO CEARÁ

34

CAEN
PÓS-GRADUAÇÃO EM ECONOMIA

SÉRIE
ESTUDOS ECONÔMICOS CAEN

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FORTALEZA • FEVEREIRO • 2019

UNIVERSIDADE FEDERAL DO CEARÁ
PROGRAMA DE PÓS-GRADUAÇÃO EM ECONOMIA - CAEN

SÉRIE ESTUDOS ECONÔMICOS – CAEN
Nº 34

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FORTALEZA – CE
FEVEREIRO – 2019

Educational Supply Policies: Distortions and Labor Market Performance*

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March 7, 2019

Abstract

Is it always worth implementing an open enrollment policy? And implementing policies that pursue equity in school supply? What is the impact of these two policies on the labor market? Do they produce efficient outcomes? This paper theoretically provides answers to these questions by studying the link between distortionary school supply policies and labor market performance. We build a two-sector labor market matching model, where the skilled segment of the economy is composed of workers who differ in the quality of the school they attended. We show the impact of government interventions to eliminate educational supply policy distortions within this theory. We demonstrate that both open enrollment and school equity policies have ambiguous effects on the labor market. Whenever their impact on the measure of workers choosing to become better educated is stronger than the additional school quality gains generated by the policy, the effects on the economy are negative. We also study the central planner solution, emphasizing the existing inefficiencies.

Keywords: Search Frictions; Human Capital; Public Policy; Composition Effect; Economic Development.
JEL Classification: J24; J31; J38; O11

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*We thank Michele Boldrin, Etienne Lehmann, Federico Etro, Fernanda Estevan, Marcelo Silva, Cézar Santos and seminar participants at PIMES-UFPe and 37th SBE Annual Meeting, Brazil.

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"Now, as a nation, we don't promise equal outcomes, but we were founded on the idea everybody should have an equal opportunity to succeed. No matter who you are, what you look like, where you come from, you can make it. Where you start should not determine where you end up."

— Barack Obama

1 Introduction

Different arguments have been proposed in the literature to explain the phenomenon of low and dispersed educational performance and the high dropout rates in some economies. Eckstein and Wolpin (1999), for instance, advocate that the decision to attend school is the result of the difference between individuals' perceived payoff received from education, which is heterogeneous and uncertain, and the disutility value related to attending school. Becker (1993), in turn, states that the accumulation of human capital is the result of an individual decision that can be compared to other forms of investment. In his view, it is expected to see an overall increase in the schooling enrollment rate when the expected benefits of this investment exceed its cost. In this way, the cheaper the schooling activity, the higher the participation rate. Becker also sustains that policies seeking to increase the option value of education or policies designed to reduce schooling costs are all expected to affect workers' investment decisions.

A different explanation for the widespread incidence of low aggregate levels of school investment was proposed by Galor and Moav (2004). The authors argue that if individual returns to education are represented by a concave function, a more effective way to increase the aggregate stock of human capital and enhance productivity is to increase school investment targeted at the less educated agents of the economy. According to them, whenever the current school quality distribution is more unequal than the social welfare distribution it is advantageous to provide more equitable education[1].

At least two points emerge from this discussion. First, any education policy must be designed considering the general equilibrium effects that exist between education and the labor market. If these links exist and are not taken into account, the impact of a policy designed to increase the size of the educated sector can generate unexpected effects on the labor market and thus change the relative returns from educational investments. Finally, the aggregate school quality distribution and the

[1]See Galor (2011) and Sauer and Zagler (2014) and references therein for a further discussion of this point.

distribution of school quality provision may affect individual decisions to study.

One of the most controversial issues in education, and closely related to the last point, concerns how the government should provide public education. Should the public school quality distribution be degenerated so as to provide everyone with the same quality of education? If this distribution is not uniform, who should receive the best school places in the economy? What is the impact of these educational distortions on the labor and education markets?

According to Jencks (1988), the public educational supply policy may be designed in many different ways. It may be conceived as an egalitarian and democratic system that gives equal opportunities to all individuals in the economy, regardless of their social and racial origin or differences in their economic background. Alternatively, it can be designed from a moralistic and compensatory view, giving students with worse initial conditions the best school choices available in the economy or even from an opposite perspective, giving the best students the best school options. Finally, from a utilitarian perspective, the educational system may also be designed in such a way that individuals have full freedom to apply for all school options available in the economy[2]. Thus, only the set of individuals who value education the most will choose the best schooling options.

Despite the noticeable attractiveness of fair educational systems, where, in theory, social and economic contexts have no impact on the quality of the school attended, what is observed is the uneven provision of school quality within and between countries. Individuals with high per-capita income or living in richer or better-managed regions often have access to the best school, while those with inferior socioeconomic background study at the worst schools[3].

Table 1 presents evidence on the previous point. Notice that in countries with a recognized unequal public educational system, such as Peru and Brazil, an improvement in the individual economic and social status implies a significant increase in average performance on the PISA educational test. In the case of Australia, for instance, this implies an increase of 44 points in the exam. It can also be seen that the Pisa test performance ratio among individuals with higher socioeconomic status and those with the lowest level easily exceeds 20% in many economies, indicating that the socioeconomic status has significant impacts on academic performance. The last indicator, percentage of resilient students, measures the proportion of individuals

[2]For more on this discussion see Coleman et al. (1966), Lazenby (2016), Gordon (2017) and references therein.

[3]Schutz, Ursprung, and Woessmann (2008) created an index of equality of educational opportunities for 54 countries. The authors show there are considerable variations in the educational equity index among OECD countries, with the U.S. being among the 25% most unequal. See Woessmann (2016) for more recent evidence.

Country	Mean Performance	Score-Point Difference	Resilient Students(%)
Australia	1.23	44	33
Belgium	1.22	48	27
Canada	1.15	34	39
Italy	1.13	30	27
Spain	1.18	27	39
U.S.	1.22	33	32
OECD	1.20	38	29
Brazil	1.21	27	9
Peru	1.34	30	3
Russia	1.09	29	26

Table 1: Mean Performance by Socioeconomic Status, 2015, PISA OECD.

Note: Mean Performance is the ratio between the mean performance of individuals in the top quarter and the bottom quarter socioeconomic status. Score-Point Difference is the mean performance gain in science associated with a one-unit increase in the index of economic, social and cultural status. A student is resilient if at the bottom quarter of economic, social and cultural status and at the top quarter of students' performance.

from the bottom of the socioeconomic distribution that are at the top of the students performance distribution. The table shows that in Spain and Canada, for example, almost 39% of all students at the bottom of the socioeconomic distribution are at the top of the achievement distribution, while in Peru only 3% of low socioeconomic students have the highest scores[4].

The link between the heterogeneous supply of local-based public goods and different socioeconomic conditions has emerged to explain the inequality of educational outcomes observed between regions and individuals with different characteristics[5]. According to this view, if individuals value school quality and this is a local public good, only those agents with greater financial conditions can afford to live in regions close to the best schools[6]. Now, since these optimal residential decisions lead to the agglomeration of individuals with the best socioeconomic background in places with the best schools, there is a positive externality acting in those regions. This externality generates an additional increase in the quality of these better schools, leading to greater educational inequality and the propagation of income dispersion in the economy[7].

[4]See Woessmann (2016) for additional evidence for other countries.

[5]See Banerjee, Lakshmi, and Rohini (2011), Galor (2011) and references therein for different theoretical explanations of these outcomes.

[6]Notice that this sorting mechanism emerges whenever house prices respond to this optimal residential decision. See Nguyen-Hoang and Yinger (2011) for a literature review on the capitalization effect of school qualities into house values. See Chyn (2018) and Chetty and Hendren (2018) for empirical evidence for the U.S..

[7]This literature has its origins in the contributions of Musgreave (1936), Samuelson (1954),

There is a well-known positive impact of education on human development. A society with unequal educational opportunities is characterized by low levels of economic mobility, which can lead to the intergenerational persistence of poverty and low levels of social and economic development. Card (1999), for instance, argues that an additional year of schooling is responsible for an increase in people's wages that ranges between 6% and 10%, according to their social and economic background. In turn, Castello-Climent (2010) shows that the greater the inequality in educational distribution, the lower the human capital accumulation and the economic growth rate are, while Sauer and Zagler (2014) argue that the more unequal the educational provision is, the more dispersed the income inequality will be, since human capital inequality directly affects the labor market returns from education. There is also no doubt that human capital investments reduce the incidence of social problems such as crime and health issues and indirectly improve a country's political and financial institutions[8]. Thus, understanding the effective mechanisms to combat inequality of educational opportunities and finding ways to increase equity and efficiency in school supply have become important topics in economic debates.

An alternative that has been widely suggested to combat the inequality of educational opportunities is the public school choice policy, considering that, in case of oversubscription, the assignment of students to school places occurs through a non-selective and biased process, such as admission lotteries[9]. Its proponents argue that since it allows parents to freely choose their children's school from all available schools in the economy, including those outside their residence area, it implies a reduction in barriers that may reduce school enrollments and aggregate students' performance.

According to Lee (1997) and Barseghyan, Clark, and Coate (2014), this policy brings at least three benefits to the education system. First, by inducing competition among schools, it generates quality and efficiency gains to the whole school system. In this perspective, some authors argue that the fiercer the school competition is to retain and attract additional students, the greater will be the gains in terms of

Tiebout (1956), Edel and Sclar (1974) and Brueckner (1979). See also Epple, Filimon, and Romer (1984), Benabou (1996a), Benabou (1996b), Fernandez and Rogerson (1996), Nechyba (1997) and Nechyba (2003) on this.

[8]See for instance Haveman and Wolfe (1984), Acemoglu and Angrist (2000) and Sianesi and Reenen (2003) for more.

[9]Belonging to this group are open enrollment policies, school vouchers, magnet and charter schools. See Reback (2008), Deming (2014) and Abdulkadiroglu, Pathak, and Walters (2018) on the impacts of school vouchers, open enrollment policies and charter schools on student achievement in the United States. In turn, Park, Shi, Hsieh, and An (2015) present evidence on the impact of magnet schools on students' performance in China. Machin and Salvanes (2016) evaluate the impact of an open enrollment educational policy reform in Norway. See Hoxby (2003) for an extensive literature review on school choice programs.

educational quality. A second advantage of this policy is that as it increases the range of school options available to each family, it is expected to cause an increase in school enrollment. Finally, one last advantage of this policy is to promote equity in the educational system, allowing everyone to have the same initial opportunities and thus compete equally for better economic prospects.

The use of admission lotteries as a mechanism to solve the problem of oversubscription to elite schools also has its praise and criticism[10]. According to Stone (2013), it is a fair and impartial mechanism, allowing any student to have the same possibility of joining an elite school. However, this advantage is also, for many, a criticism. If the volume of students queuing for some school places is extensive, what is the impact of selecting them that way? Isn't the efficiency of the whole education system being degraded with this policy? This work examines these questions, suggesting an answer to the impacts of educational supply policy distortions on the education and labor markets. Specifically, we study the effects of an open enrollment policy and the elimination of admission lotteries on the individual decisions to study and their impacts on the labor market.

In our model, there are two productive sectors: educated and non-educated[11]. All agents are identical, except for their educational costs. An individual who decides to study works exclusively in the qualified sector and the set of school vacancies available in the economy is heterogeneous with respect to quality. Following the previous evidence, we assume that the public school quality is supplied through a policy that assigns the best school places to individuals with the lowest educational costs and the worst school places to individuals with the highest schooling cost in the economy, and when there is oversubscription, the admission decision is made through lotteries[12].

We show, in a general equilibrium model with endogenous schooling decisions and labor market frictions, that these two policies generate ambiguous results. In particular, an open enrollment policy generates an increase in the mass of educated workers, but has ambiguous effects on their quality. Whenever the policy impact on the aggregate demand for education is stronger than the additional school quality gains, there is a reduction in the average quality of the educated workforce. This reduction, in turn, generates an additional distributional effect in the labor market, by reducing the relative attractiveness of the skilled sector.

[10]See Stone (2008) for a literature review on the use of lotteries as a school admission mechanism. Abdulkadiroglu, Angrist, Hull, and Pathak (2016) present evidence of the impact of lotteries on students' performance.

[11]We call these two sectors interchangeably as skilled and unskilled or qualified and non-qualified.

[12]We mean, with this assumption, that there is a random assignment of school places to students in case of oversubscription. We come back to this point later.

Assuming that there is an increase in the quality of the skilled labor force after the policy implementation, this implies an increase in the wage rate and greater job creation dynamics in this segment. The impact on skilled sector unemployment is ambiguous, as there is also an increase in the measure of skilled agents in the economy. These same results arise if we eliminate the admission lottery policy. However, it now generates strong composition effects. The skilled labor force is now composed of individuals with the highest educational costs, while the unskilled workforce is made up of individuals with the lowest costs of schooling in the economy.

In terms of welfare, we find that the use of admission lotteries generates inefficient decentralized outcomes. There is overemployment in the unskilled sector. In turn, we detect that whenever this admission policy induces an excess of demand for education, so that it exceeds the efficient measure of educated individuals, there is underemployment in the schooled sector compared to the social planner's solution.

To better understand our previous results, consider an economy with a distorted school supply policy that provides its best school places to individuals with the lowest schooling costs, and in case of oversubscription, uses admission lotteries to select students. The government then puts in place a policy that increases the set of school options available to all agents in the economy. The expansion of the school supply generates an increase in average quality of the skilled labor force. However, the government school supply policy also exerts an indirect and negative effect on the economy. Since there is an increase in the average quality of schools available to each agent, it becomes more advantageous to study. The higher education demand thus reduces the average quality of the skilled labor force and indirectly depresses the skilled sector. The final outcome depends on these two opposing effects.

Notice that the change in the average quality of the skilled labor force in our model is closely related to the shape of both the school quality and the schooling cost distributions. If the additional school quality gains are greater than the increase in the measure of students, the policy is followed by an increase in the average quality of the skilled labor force. This may happen, for instance, if the two distributions are asymmetric and the school quality distribution has negative skewness, while the schooling cost distribution has a tail to its right.

Finally, it should be mentioned that the educational and the labor markets interact on two fronts in our model. On the one hand, the increase in the average quality of the educated workforce positively affects the qualified sector, with only indirect effects on the non-qualified sector. Due to labor market imperfections, there may be an increase or a decrease in search externalities within and between workers and firms[13]. On the other hand, an expanding skilled sector encourages school investments, which leads to an increased mass of skilled workers in the economy.

Related Literature

Our model is related to a growing literature that studies the determinants of workers' investments in education and how the decentralized equilibrium compares to the social optimum outcome. On the one hand, some authors have argued that the decentralized equilibrium is characterized by a low and inefficient level of human capital investments. Thus, government interventions are necessary to reduce inefficiency, as suggested by Acemoglu (1996). On the other hand, some authors present evidence of high and inefficient levels of education investments. Charlot and Decreuse (2005), for instance, sustain that when education investments and labor market returns are positively related, the overeducation phenomenon arises. They argue that as the size of the educated workforce increases, there will be a reduction in the average skills of both the schooled and unschooled segments of the labor force, which implies a decrease in firms' incentives hire more workers. They conclude that any welfare improvement policy should be designed to deter low-skilled individuals from entering the schooled segment of the economy.

We show that this result can be found in a more general model with a distorting school supply and an endogenous demand for education. In particular, whenever the decentralized demand for education is lower than the efficient demand, the optimal policy is the one proposed by Charlot and Decreuse (2005), that is, a reduction in the mass of individuals who seek schooling.

The present model is also related to a recent contribution of Benegas and Corrêa (2017). They examine the impact of a first-order improvement in school quality distribution within the mass of educated workers and the aggregate productivity. This article shows that a policy that seeks both equity and expansion of the school supply, such that everyone can access all schools available in the school system, generates ambiguous results in the economy. On the one hand, there is a direct and positive impact on the average productivity of the educated workforce employed in the skilled sector, but there is also an indirect and negative effect that leads to a reduction in the average productivity resulting from the education demand.

There is extensive literature addressing the issue of individual heterogeneity in two-sector and imperfect labor market models with different labor and product market assumptions. Many of these works address the impacts of different levels of

[13]There is an increase in the labor market tightness in the unskilled sector, due to the smaller unskilled labor force and the absence of changes in the job creation dynamics in this segment.

market segmentation on the aggregate equilibrium and economic welfare. Blasquez and Jansen (2008) and Albrecht, Vroman, and Navarro (2010), for instance, argue that there is a negative externality generated by a higher measure of workers that self-select in education in the unskilled segment of the economy. They contend that schooling investments affect both average productivity in the skilled sector and labor market tightness in the skilled and unskilled sectors. In turn, Moen (2003) and Charlot, Malherbet, and Ulus (2013) state that the main problem related to human capital investments is not that skilled workers hurt unskilled employees, but the opposite.

There is also a recent literature that aims to explain the reasons for the recent rise in wage inequality among firms, sectors and educational levels. For instance, Acemoglu (2003) argues that this phenomena is closely related to a recent increase in the skill premium induced by skill-biased technological progress, while Altonji (2014) and Card, Heining, and Kleine (2013) suggest that to better understand the recent trend in wage inequality it is necessary to study in-group wage dispersion and the way in which firms and workers self-select their jobs.

Our paper is related to this previous literature. We show that the more dispersed the public school quality distribution is in our model, the greater is the wage inequality among individuals who decide to study. In turn, the more exclusive the schooling option is, the higher the wage inequality among groups tends to be.

The present paper is also related to the large literature that evaluates the impact of different pull and push factors on students' dropout behavior. See, for instance, Doll, Eslami, and Walters (2013); Bound, Lowenstein, and Turneri (2010) and Hanushek, Lavy, and Hitomi (2010) on the impact of school resources (push factor) on students' dropout behavior and Gustman and Steinmeier (1981) and Johnson (2013) and references therein on the impact of the main labor market variables on school enrollment.

Finally, there is also a large literature that studies the link between distortionary taxes and the provision of public goods, such as Bovenberg and Jacobs (2005), Maldonado (2008) and Stantcheva (2017). They show that government should subsidize human capital investments in order to alleviate the distortionary effect of income taxes. Our work differs from these papers in many points. For instance, we study the impacts of distorted public good provision on a frictional labor market, while those other authors study the impacts of distortionary taxes.

Besides this introduction, this paper has four more sections. In the next one we introduce the benchmark model and describe the decentralized equilibrium. The following section shows the impacts of educational policy distortions. Section 4 characterizes the social welfare allocations while the last section contains the main concluding remarks.

2 The Economy

The economy is composed of firms that once matched with workers produce of a single consumption good, whose price is normalized to one. Let the time be continuous and consider that each firm has access to a production technology with labor as the only input.

The only final consumption good can be produced by firms in both the skilled (S) and unskilled (N) segments of the economy. Consider, as Smith (1999) and Cahuc and Wasmer (2001), that the size of the labor force employed by each company is endogenous and the two sectors are segmented.

There is a measure one of heterogeneous infinitely lived individuals in the economy. They are all born with the ability to work in the unskilled segment of the economy with exogenous productivity q_L. However, agents can study at the individual cost I and become qualified to work in the skilled sector. Let $H(I)$, with support in the interval $[I_L, I_H]$, be the distribution of the individual cost of education.

In the early stages of their lives, individuals can study for an exogenous fixed period of time T. They can also work or search for a job vacancy in the unskilled sector if they decide not to study. In the remaining periods of their lives, they can only be working or unemployed and searching for a job. Consider, without loss of generality, that the labor market productivity of a skilled individual depends on the quality of the school attended, $q \in [q_L, q_H]$.

Government runs the education system. Consider there are distortions in the school supply policy and that the school system is composed of a continuum of q-quality school places defined by the distribution $G(q)$ in the support $[q_L, q_H]$. More specifically, we assume that the government does not offer all agents the same set of school vacancies. It assigns the best school places to the best students and the worst choices of schooling to the worst group of students[14].

The public education policy and individual schooling decisions take place as follows. First, the government observes the individual cost of education, $I \in [I_L, I_H]$. Then, it provides to each agent school places in the set of school qualities, $[q_L, Q_o(I)]$, where $Q_o(I)$ describes the best school quality option available to agent I. Agents then evaluate working and educational options according to their schooling cost and

[14]As previously mentioned, we are considering the worst students as the set of agents with higher schooling costs in the economy. The best students, in turn, are the group of individuals with lower educational costs.

the labor market returns from their human capital investments in a school of quality q. Let $Q_d(I)$ represent the reservation school quality that leaves an individual with schooling cost I indifferent between entering the labor force as an unskilled worker and studying. Whenever $q \geq Q_d(I)$, the individual I decides to go to school, thus becoming educated. On the other hand, if $q < Q_d(I)$, this agent decides to work in the unskilled sector, since the labor market returns are bigger than the net benefits received from schooling investments. Finally, the government uses a school lottery to define the specific school to be attended by each agent I[15].

2.1 Labor Market

Consider, as standard in the search literature, that before starting production, workers and firms are involved in a search process to find a productive partner. Let k_S and k_N ($k_S > k_N$) represent the search costs of a firm that decides to open a vacancy in the skilled and the unskilled sector, respectively.

The number of job matches formed per period is given by a non-negative, concave and homogeneous degree one matching function, $m(v_i, u_i)$, which is increasing in its arguments. Let v_i represent the vacancy rate and u_i denote the fraction of type $i = \{S, N\}$ unemployed workers in the economy. As usual, it can be shown that the probability rate of filling a vacancy is given by: $p(\theta_i) = \frac{m(v_i,u_i)}{v_i}$, where $\theta_i = \frac{v_i}{u_i}$ denotes the tightness of sector i. In turn, the rate at which an unemployed worker moves to employment status is given by $z(\theta_i) = \theta_i p(\theta_i) = \frac{m(v_i,u_i)}{u_i}$.

Firms

Production can be performed by firms in the two sectors. Assume that:

$$F_N(l_N) = q_L l_N^{\alpha_N}, \tag{1}$$

$$F_S(l_S) = q^e(Q_d, Q_o) l_S^{\alpha_S}, \tag{2}$$

represent the production technologies used in the unskilled and skilled sectors, respectively. The terms q_L and $q^e(Q_d, Q_o)$ represent workers' productivity in the

[15]Notice that the evidence that the public supply of school vacancies is not homogeneous, especially among individuals with different educational costs, and that the government uses lotteries as a mechanism for assigning school places to students in case of excessive subscription is abundant in the literature. See for instance Benabou (1996a), Benabou (1996b), Fernandez and Rogerson (1996), Nechyba (2003), Levitt (2006), Staiger (2014), Zhang (2016) and references therein on these two points.

unskilled and the skilled sector, respectively[16]. Let $\alpha_i \in (0, 1]$.

Notice from the two previous expressions that the production functions have decreasing returns to the amount of labor employed in each sector[17]. The higher the quality of the schooled labor force, the bigger the value of $q^e(Q_d, Q_o)$ and the average productivity in the skilled sector are. In turn, the larger the number of skilled workers employed in the skilled sector, the lower is the marginal contribution of each additional worker to the aggregate production in this sector.

Firms decide whether they enter each productive sector or not and their optimal number of vacancies. The following Hamilton-Jacobi-Bellman equations describe the problem of a representative firm in each sector[18]:

$$\rho\Pi_N(l_N) = \max_{v_N}\{q_L l_N^{\alpha_N} - w_N(l_N)l_N - k_N v_N - C_N + \frac{\partial \Pi_N(l_N)}{\partial l_N}[p(\theta_N)v_N - \lambda_N l_N]\}, \quad (3)$$

$$\rho\Pi_S(l_S) = \max_{v_S}\{q^e(Q_d, Q_o) l_S^{\alpha_S} - w_S(l_S)l_S - k_S v_S - C_S + \frac{\partial \Pi_S(l_S)}{\partial l_S}[p(\theta_S)v_S - \lambda_S l_S]\}. \quad (4)$$

Equations (3) and (4) have similar interpretations. We focus first on (3). It tells us that a firm matched with l_N workers of quality q_L produces $F_N(l_N)$ units of the final consumption good per period. The firm pays $w_N(l_N)$ as the unskilled wage rate and C_N, as a fixed production cost. To open a vacancy, any given company in the unskilled sector must spend k_N, as search costs.

The final terms in equation (3) are related to the flow of workers between employment and unemployment status. This flow is defined by: $\dot{l_N} = p(\theta_N)v_N - \lambda_N l_N$, where the first element on the right-hand side is related to the rate at which each vacancy becomes occupied. The second term determines the flow of workers that lose jobs in each time period.

The second equation resembles the first one. The main differences can be found in the terms related to the production function and wage rate equation. A firm matched with l_S workers of average quality $q^e(Q_d, Q_o) = E[q \mid Q_d \leq q \leq Q_o]$ produces $F_S(l_S)$ and pays $w_S(l_S)$ to each worker. To open a skilled vacancy, each firm must spend k_S. As before, the final terms in equation (4) are related to the flow of workers between employment and unemployment statuses.

[16]Note that the average productivity of the skilled sector depends on both aggregate demand (Q_d) and supply of education (Q_o). We return later, in the aggregation subsection, to these variables.

[17]The assumption of decreasing returns is becoming more and more common in search theory. See, for instance, Kaas and Kircher (2015) and references therein on this point.

[18]Note that these are the stationary versions of the Hamilton-Jacobi-Bellman equations. We assume this simplified version because we are only interested in the steady state equilibrium of our model economy.

The set of conditions that characterize the optimal firm decisions are given by[19]:

$$k_N - \frac{\partial \Pi_N(l_N)}{\partial l_N} p(\theta_N) = 0, \tag{5}$$

$$k_S - \frac{\partial \Pi_S(l_S)}{\partial l_S} p(\theta_S) = 0, \tag{6}$$

$$\begin{aligned} \rho \frac{\partial \Pi_N(l_N)}{\partial l_N} &= \alpha_N q_L l_N^{\alpha_N - 1} - w_N(l_N) - w_N^{'}(l_N) l_N \\ &- \frac{\partial \Pi_N(l_N)}{\partial l_N} \lambda_N + \frac{\partial^2 \Pi_N(l_N)}{\partial l_N^2} [p(\theta_N) v_N - \lambda_N l_N], \end{aligned} \tag{7}$$

$$\begin{aligned} \rho \frac{\partial \Pi_S(l_S)}{\partial l_S} &= \alpha_S q^e(Q_d, Q_o) l_S^{\alpha_S - 1} - w_S(l_S) - w_S^{'}(l_S) l_S \\ &- \frac{\partial \Pi_S(l_S)}{\partial l_S} \lambda_S + \frac{\partial^2 \Pi_S(l_S)}{\partial l_S^2} [p(\theta_S) v_S - \lambda_S l_S]. \end{aligned} \tag{8}$$

In the steady state, we also have that:

$$\dot{l_N} = \dot{l_S} = 0. \tag{9}$$

By using expressions (5) and (6), together with equation (9), in (7) and (8), we arrive at[20]:

$$\frac{k_N(\rho + \lambda_N)}{p(\theta_N)} = \alpha_N q_L l_N^{\alpha_N - 1} - w_N(l_N) - w_N^{'}(l_N) l_N, \tag{10}$$

$$\frac{k_S(\rho + \lambda_S)}{p(\theta_S)} = \alpha_S q^e(Q_d, Q_o) l_S^{\alpha_S - 1} - w_S(l_S) - w_S^{'}(l_S) l_S. \tag{11}$$

These two equations determine the equilibrium values of θ_N and θ_S, characterizing the equilibrium labor demand. Focusing first on equation (10), the left-hand side represents the expected cost of occupying a vacancy in the unskilled sector. The other side of the expression is related to the expected profit associated with the creation of an additional vacancy. The equilibrium value of θ_N is established in order to equate these two expected returns. The left-hand side of the second

[19]Expressions (5) and (6) are the first-order conditions for v_N and v_S. The expressions that follow are the envelope conditions for l_N and l_S. v_i and l_i, for $i = \{N, S\}$, are the control and the state variables, respectively.

[20]Notice that $w_N^{'}(l_N) = \frac{\partial w_N(l_N)}{\partial l_N}$ and $w_S^{'}(l_S) = \frac{\partial w_S(l_S)}{\partial l_S}$.

expression represents the expected cost of creating a type S vacancy. The other side of this expression is related to the benefits of an additional vacancy. It is important to point out that an increase in the wage rate or a fall in $q^e(Q_d, Q_o)$ both come with a decrease in θ_S.

The usual hypothesis of free entry and exit conditions assures that in equilibrium, current corporate profits are nil. Then,

$$q_L l_N^{\alpha_N} - C_N - w_N(l_N) l_N = \frac{k_N \lambda_N l_N}{p(\theta_N)}, \tag{12}$$

$$q^e(Q_d, Q_o) l_S^{\alpha_S} - C_S - w_S(l_S) l_S = \frac{k_S \lambda_S l_S}{p(\theta_S)}. \tag{13}$$

The left-hand sides of these two equations are associated with the firm revenues while the right-hand sides give us the firm costs.

Workers

Let $W_N(l_N)$ and U_N($W_S(l_S)$ and U_S) be the present discounted value of the expected gains associated with employment and unemployment statuses for an unskilled (skilled) worker. An unemployed worker with schooling cost I who has studied receives b_S units of the consumption good as unemployment benefits per period. At an instantaneous rate $z(\theta_S)$, the educated unemployed worker finds a vacant job, moving to employment status[21]. In this way we have that:

$$\rho W_N(l_N) = w_N(l_N) - \lambda_N(W_N(l_N) - U_N), \tag{14}$$

$$\rho U_N = b_N + z(\theta_N)(W_N - U_N), \tag{15}$$

$$\rho W_S(l_S) = w_S(l_S) - \lambda_S(W_S(l_S) - U_S), \tag{16}$$

$$\rho U_S = b_S + z(\theta_S)(W_S(l_S) - U_S), \tag{17}$$

determine the value functions of a non-educated and an educated worker, respectively employed and unemployed. These expressions are standard in the search literature. The first equation implies that a non-educated worker employed in a firm with l_N unskilled workers receives $w_N(l_N)$ units of the consumption good as wages per period. This job position is destroyed due to an idiosyncratic shock that occurs at rate λ_N. Expression (15), in turn, indicates that an unskilled worker

[21]Workers who do not study receive b_N units of the consumption good as unemployment insurance and they move to employment status at a rate $z(\theta_N)$.

receives b_N as unemployment benefit per period. At rate $z(\theta_N)$, this unemployed worker finds a job vacancy, thus moving to employment status.

If a particular match is destroyed, both the worker and the firm have to pay the costs related to the return to the search process. In this way, a productive match generates a surplus that has to be distributed between the two parties. Consider, as usual in job search theory, that this division is determined by the Generalized Nash Bargain Solution between the firm and the worker, where β_i represents workers' bargaining power in sector $i = \{S, N\}$. The wage rates satisfy:

$$\beta_N \frac{\partial \Pi_N(l_N)}{\partial l_N} = (1 - \beta_N)[W_N(l_N) - U_N], \tag{18}$$

$$\beta_S \frac{\partial \Pi_S(l_S)}{\partial l_S} = (1 - \beta_S)[W_S(l_S) - U_S]. \tag{19}$$

Using expressions (7) - (8) and (14) - (19), the wage rates are respectively given by[22]:

$$w_N(l_N) = \frac{\beta_N \alpha_N q_L}{\beta_N \alpha_N + (1 - \beta_N)} l_N^{(\alpha_N - 1)} + \rho(1 - \beta_N) U_N, \tag{20}$$

$$w_S(l_S) = \frac{\beta_S \alpha_S q}{\beta_S \alpha_S + (1 - \beta_S)} l_S^{(\alpha_S - 1)} + \rho(1 - \beta_S) U_S.$$

These two expressions give us the wage rates at the two sectors. Notice that the wage rates are a weighted sum of two terms. The first one is related to workers' job match productivity and the other to the workers' outside options. Since job match productivity varies, depending on whether the workers are educated or not and on the quality of the school attended, the first term differs between the two types of workers. Therefore, the higher the quality of the school attended, the bigger the job match productivity and the skilled wage rate will be[23].

The average wage rate in the skilled sector is given by:

$$w_S^e(l_S) = \frac{\beta_S \alpha_S q^e(Q_d, Q_o)}{\beta_S \alpha_S + (1 - \beta_S)} l_S^{(\alpha_S - 1)} + \rho(1 - \beta_S) U_S. \tag{21}$$

[22]Notice that the bargaining between workers and firms is on the marginal surplus generated by the additional worker. See Pissarides (2000) on this point.

[23]Considering the assumption of strict concavity of the production function, it can be shown that workers' productivity decreases while the number of employed workers increases. Thus, there is a reduction in the marginal cost of an additional worker, as firms become larger. See Smith (1999).

2.2 Schooling Sector

We have previously defined the optimal demand and supply of labor in each productive sector. Now we characterize the demand and supply of education. As previously mentioned, the individual decision to study is made by comparing the net benefits of investing in education with the returns obtained in the unskilled sector. Let:

$$\int_0^\infty e^{-\rho t}\rho U_N \, dt,$$

represent the present value of gains related to early entry into the labor force as a non-schooled worker. However, if someone with schooling cost $I \in [I_L, I_H]$ decides to study, the discounted present value of such decision is given by:

$$\int_0^T -e^{-\rho t}\rho I q(I) \, dt + \int_T^\infty e^{-\rho(t-T)}\rho U_S \, dt,$$

where the first term is related to the individual schooling costs materialized during the exogenous compulsory period of studies T. The following term refers to the benefits of being an educated worker. Notice that the investment in education depends both on individual and aggregate variables. The cost of education is agent-specific and depends on I and q. However, the returns depend on the aggregate schooling decisions, which define the labor market returns of being an educated worker, $q^e(Q_d, Q_o)$. From the two previous expressions we have that whenever:

$$\int_0^T -e^{-\rho t} I q(I) \, dt + \int_T^\infty e^{-\rho(t-T)} U_S \, dt \geq \int_0^\infty e^{-\rho t} U_N \, dt,$$

the individual I decides to study.

Assume that $Q_d(I) \in (q_L, q_H)$, for all schooling costs[24], represents the value of $q(I)$ that balances the two sides of the previous expression. It determines the lowest value of school quality that leaves individual I indifferent between studying or not. Then, we have that:

$$-\frac{(1-e^{-\rho T})}{\rho} I Q_d(I) + \frac{U_S}{\rho} = \frac{U_N}{\rho}.$$

Rearranging this expression gives the school quality reservation value that leaves

[24] This condition excludes the following limit cases: only individuals with the lowest cost of education will study, $Q_d(I_L) = q_H$, and all agents decide to study, $Q_d(I_H) = q_L$.

individual I indifferent between studying and working in the first T periods of life:

$$Q_d(I) = \frac{U_S - U_N}{(1 - e^{-\rho T})I}. \tag{22}$$

This last expression establishes the minimum public school quality compatible with the indifference between studying and working in the unskilled sector. It characterizes the demand for education of an agent $I \in [I_L, I_H]$[25]. Notice that if the school offered by the government has quality defined in the set $[Q_d(I), q_H]$, the individual will always study, becoming a skilled worker after T periods. However, if the school quality received is in the set $[q_L, Q_d(I))$, individual I will not study. Figure 2 shows the demand for education.

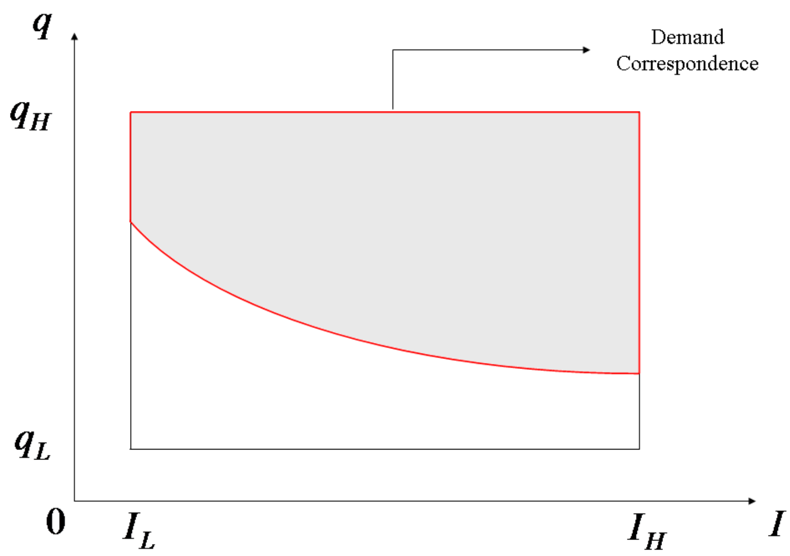

Figure 1: Demand Correspondence, $Q_d(I)$

Notice that for each individual I there is a set of acceptable school options. For instance, an individual with the highest schooling cost in the economy, I_H, accepts a larger set of school vacancies than his counterparts. This set is represented by the vertical segment in red at I_H. The grey area in Figure 2 shows the demand correspondence in the economy.

[25]Notice that $Q_d(I)$ is continuously differentiable in $[I_L, I_H]$ and $Q'_d(I) < 0$ and $Q''_d(I) > 0$. Also, the higher the skilled unemployment option, $U_S(q^e(Q_d, Q_o))$, the bigger $Q_d(I)$ is.

Now consider the education supply. As previously mentioned, there are distortions in the education supply policy. More specifically, we assume that government defines the set of school option available to each agent I, $[q_L, Q_o(I)]$. Once this set has been defined, each agent is randomly assigned to a particular school in this set, as established the admission lottery policy. Thus, let the schooling policy be defined by:

$$Q_o(I) = q_H - (q_H - q_L)(\frac{I - I_L}{I_H - I_L})^{\epsilon}, \tag{23}$$

where $\epsilon \in [1, \infty)$. Notice from (23) that as ϵ increases, the school quality term $Q_o(I)$ and the set of school options available to individual I, $[q_L, Q_o(I)]$, become greater. Thus, we can see this term as a school policy parameter that defines, for each individual I, the set of schools to attend. Note that individuals with a higher school cost will have at their disposal a set of schools with lower average quality than other individuals with lower school costs. It can also be shown that the individual with the highest schooling cost in the economy, I_H, receives the lowest school quality available, $Q_o(I_H) = q_L$. In turn, the agent with the lowest schooling cost, I_L, receives the best school option available in the economy, $Q_o(I_L) = q_H$. Figure 3 characterizes the supply of education in the economy. It also presents the equilibrium in the schooling sector.

In order to better understand the educational decisions, consider a particular agent with schooling cost $\bar{I} \in [I_L, I_H]$. The term $Q_o(\bar{I})$ in Figure 3 represents the best school quality option provided by the government to this agent. In turn, $Q_d(\bar{I})$ characterizes the minimum school quality compatible with individual $\bar{I}$'s decision to become schooled. Note that, given the admission lottery policy and the optimal individual decisions of schooling, the student may study at any school in the subset $[Q_d(\bar{I}), Q_o(\bar{I})]$. An example of a randomly assigned school is $\bar{q}$.

It can also be seen in Figure 3 that there is a unique $\tilde{I}$ that characterizes the set of skilled workers in the economy, $Q_o(\tilde{I}) = Q_d(\tilde{I})$, and settles the equilibrium average quality of the educated workforce in the economy. If $I \leq \tilde{I}$, then $Q_d(I) \leq Q_o(I)$ and the agent with cost I will become schooled. However, if $I > \tilde{I}$, then $Q_d(I) > Q_o(I)$. In this last case, the school offer does not meet the minimum individual quality requirement to study, $Q_d(I)$, so the agent decides to become unskilled. In short, an individual with schooling cost I will have ability given by:

$$q(I) \begin{cases} \in [Q_d(I), Q_o(I)] & \text{if } I \leq \tilde{I} \\ = q_L & \text{otherwise.} \end{cases}$$

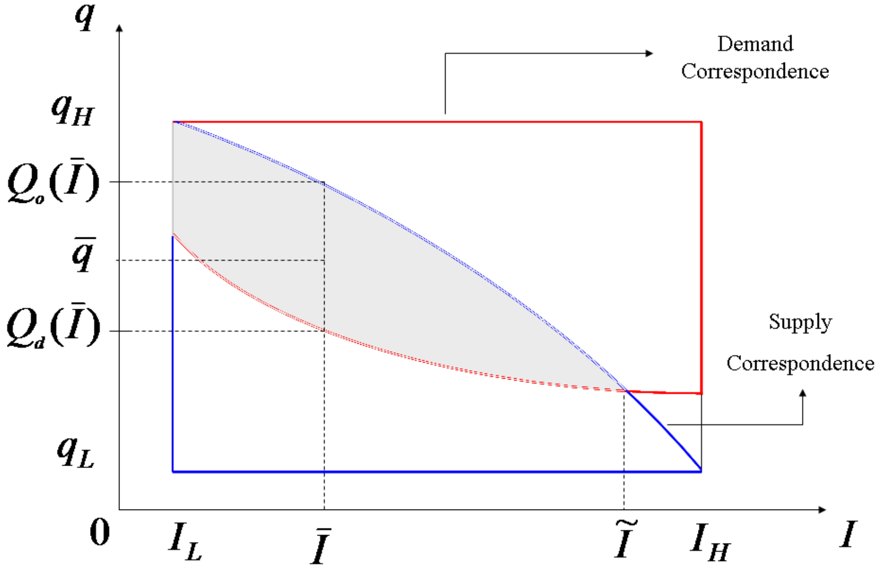

Figure 2: Schooling Sector Equilibrium

2.3 Aggregation and the Decentralized Equilibrium

Now we formally characterize the aggregate productivity in the skilled sector, $q^e(Q_d, Q_o)$. Let Q_d and Q_o represent the aggregate school quality demanded by individuals and the aggregate school quality supplied by the government, respectively. They are respectively given by:

$$Q_d = E[Q_d(I) \mid I \leq \tilde{I}] = \int_{I_L}^{\tilde{I}} \frac{U_S - U_N}{(1 - e^{-\rho T})I} \frac{dH(I)}{H(\tilde{I})}, \tag{24}$$

$$Q_o = E[Q_o(I) \mid I \leq \tilde{I}] = q_H - (q_H - q_L) \int_{I_L}^{\tilde{I}} (\frac{I - I_L}{I_H - I_L})^{\epsilon} \frac{dH(I)}{H(\tilde{I})}. \tag{25}$$

Expression (24) presents the aggregate minimum quality demanded by all individuals who decide to study in our economy. Notice that it equals the sum of all individual demands for education weighted by the density of agents that study, $\frac{h(I)}{H(\tilde{I})}$, for all $I \in [I_L, \tilde{I}]$. The following equation characterizes the aggregate maximum school quality supplied by the government weighted again by the density of agents that study. Note that once these two aggregate quantities are found, it is possible to obtain the average productivity of the skilled segment of the economy, $q^e(Q_d, Q_o) = E[q \mid Q_d \leq q \leq Q_o]$.

Expressions (24) and (25) also deserve some additional comments. Let μ represent the mean of the school quality distribution, $G(q)$. It can be shown that:

(i) $\lim_{Q_d \to q_H} q^e(Q_d, Q_o) = q_L$, if $Q_o = q_L$;

(ii) $\lim_{Q_d \to q_L} q^e(Q_d, Q_o) = \mu$ and $\lim_{Q_d \to q_H} q^e(Q_d, Q_o) = q_H$, if $Q_o = q_H$[26].

These two results guarantee there will be no schooled sector in the economy if the public supply of education is given by $Q_o = q_L$, $\forall I$. They also assure that if $Q_o = q_H$, the average productivity of the skilled sector converges to the mean of the school quality distribution, μ, if the aggregate demand of education is q_L. In turn, as the size of the non-educated labor force converges to the unit, the average productivity of the educated workforce moves to the highest value of the distribution, q_H[27]. Another interesting aspect of the skilled labor force is:

$$\frac{\partial q^e(Q_d, Q_o)}{\partial Q_d} = \frac{q^e(Q_d, Q_o) - Q_d}{G(Q_o) - G(Q_d)} g(Q_d) > 0;$$

$$\frac{\partial q^e(Q_d, Q_o)}{\partial Q_o} = \frac{Q_o - q^e(Q_d, Q_o)}{G(Q_o) - G(Q_d)} g(Q_o) > 0;$$

since $Q_d < q^e(Q_d, Q_o) < Q_o$. Thus, any policy that reduces the aggregate demand for education (i.e. increases Q_d) implies a rise in $q^e(Q_d, Q_o)$. In turn, any policy that reduces the education supply implies the opposite effect.

Notice that the returns to education in our model are subject to negative agglomeration externalities. The greater the number of individuals who decide to study, the lower the expected return on education and the aggregate productivity of the skilled sector will be.

An important point to investigate is the impacts of a policy that expands school quality provision on the quantity and quality of the educated workforce. An answer to this point can be found through changes of the distorting parameter ϵ.

Proposition 1 *Consider an increase in the policy parameter ϵ. It implies:*

(i) A reduction in Q_d;

(ii) An increase in Q_o.

[26] We only need to apply L'Hopital's Rule to prove these results.

[27] Consider that the schooling distribution is non-degenerate such that $\mu < q_H$. This result can be used to explain the stylized fact that countries with low education levels tend to pay higher wage rates to their educated workforce. See Avalos and Savvides (2006), Birdsall, Ross, and Sabot (1995), Bils and Klenow (2000), Psacharopoulos and Patrinos (2004) and references therein on this topic.

Proof. See Appendix A. ■

The previous proposition guarantees that a higher provision of educational quality - through a higher value of ϵ - implies both an increase in demand for and supply of education. The latter impact is direct, whereas the former one is indirect and occurs through a higher equilibrium value of $\tilde{I}$. Note that the higher the increase in ϵ the lower the equilibrium value of Q_d and the greater the aggregate demand for education. Q_o also increases with higher ϵ. Therefore, the policy that improves the set of school qualities available to each agent has an ambiguous impact on the average quality of the skilled labor force, $q^e(Q_d, Q_o)$. On the one hand, it increases the average quality of the skilled labor force through a high value of Q_o. On the other hand, it decreases the average quality $q^e(Q_d, Q_o)$, through a smaller value of Q_d.

Definition 2 *A steady-state block equilibrium is characterized by a thirteen-tuple:* $(\theta_i, v_i, l_i,\ w_i(\cdot), u_i, Q_d, Q_o, \tilde{I})$ *such that:*

(i) $\rho U_i = b_i + \frac{\beta}{1-\beta} k_i \theta_i,\ \theta_i = \frac{v_i}{u_i}$ *and* $p(\theta_i) v_i = \lambda_i l_i$*, for* $i = \{S, N\}$;

(ii) $Q_o(\tilde{I}) = Q_d(\tilde{I})$;

(iii) *equations* (10), (11), (12), (13), (20), (21), (24) *and* (25) *are satisfied.*

The equilibrium has a block recursive structure. First, given the distributions of $G(q)$ and $H(I)$, the government determines the equilibrium value of Q_o. Then, individuals determine the aggregate demand of education, Q_d, and firms set the equilibrium labor demand for each sector. The remaining labor market equilibrium variables follow.

Notice that using expressions (10) and (20), we obtain the equilibrium values of $w_N(\cdot)$ and θ_N. It can be seen that the equation that characterizes the equilibrium value of θ_N does not depend on θ_S or on $q^e(Q_d, Q_o)$. By using similar reasoning, equations (21) and (11) determine the equilibrium value of $w_S^e(\cdot)$ and θ_S. They are both functions of $q^e(Q_d, Q_o)$. The expressions that characterize the equilibrium values of θ_N, θ_S, Q_d and Q_o are respectively given by:

$$\frac{k_N[\rho + \lambda_N + \beta_N z(\theta_N)]}{p(\theta_N)} = \left\{ \frac{(1-\beta_N)(1-\alpha_N)^{(1-\alpha_N)}}{[(1+\rho)C_N]^{(1-\alpha_N)}[\beta_N \alpha_N + (1-\beta_N)]} \right\}^{\frac{1}{\alpha_N}} \alpha_N q_L^{\frac{1}{\alpha_N}} - (1-\beta_N) b_N;$$

$$\frac{k_S[\rho + \lambda_S + \beta_S z(\theta_S)]}{p(\theta_S)} = \left\{ \frac{(1-\beta_S)(1-\alpha_S)^{(1-\alpha_S)}}{[(1+\rho)C_S]^{(1-\alpha_S)}[\beta_S \alpha_S + (1-\beta_S)]} \right\}^{\frac{1}{\alpha_S}} \alpha_S q^e(Q_d, Q_o)^{\frac{1}{\alpha_S}} - (1-\beta_S) b_S;$$

$$Q_d = \frac{(b_S + \frac{\beta_S}{1-\beta_S} k_s \theta_S) - (b_N + \frac{\beta_N}{1-\beta_N} k_n \theta_N)}{\rho(1-e^{-\rho T})} \int_{I_L}^{\tilde{I}} \frac{1}{I} \frac{dH(I)}{H(\tilde{I})};$$

$$Q_o = q_H - (q_H - q_L)\int_{I_L}^{\tilde{I}} (\frac{I - I_L}{I_H - I_L})^{\epsilon} \frac{dH(I)}{H(\tilde{I})}.$$

The first two expressions characterize the job creation dynamics in the unskilled and skilled segments of the economy[28]. The left-hand side of these two expressions gives the costs of opening an additional vacancy in each sector. In turn, the right-hand side characterizes the benefits of this new job vacancy. In equilibrium, the two sides of the expression must be equal. The following expressions characterize the aggregate demand and supply of education, respectively. The first one is obtained after substituting the expressions that characterize the option value of unemployment in both sectors in equation (24). The next expression is equation (25).

3 The Impact of Educational Policy Distortions on the Labor Market

We are now in a position to evaluate the impact of the two educational supply policy distortions on the decentralized equilibrium. In particular, we are interested in assessing the impact of the two educational supply policies on the skilled sector labor productivity, job creation flows, wage and the unemployment rates in both the skilled and the unskilled segments of the economy.

Consider initially the first distortion. Notice that we could see this policy (when $\epsilon \to \infty$) as one instituted by an egalitarian government that does not discriminate among agents and offers to all of them the same set of school options, $[q_L, q_H]$[29]. The following proposition presents the impact of this policy on the aggregate productivity of the skilled sector.

Proposition 3 *Let* $[Q_o - q^e(Q_d, Q_o)]g(Q_o) > [q^e(Q_d, Q_o) - Q_d]g(Q_d)$. *Thus,*

$$\frac{d}{d\epsilon} q^e(Q_d, Q_o) > 0.$$

In particular,

$$\lim_{\epsilon \to \infty} q^e(Q_d, Q_o) > q^e(Q_d, Q_o),$$

[28]They can be obtained from substituting the wage rate equations in (10) and (11) and in (12) and (13). We then manipulate these derived expressions and consider the Nash bargaining conditions.

[29]Notice that in this first analysis, we maintain the assumption that there is a lottery that randomly assigns individuals to schools in this extended set. Next, we evaluate the impact of this friction.

for any values of Q_d and Q_o in $[q_L, q_H]$.

Proof. See Appendix B. ■

Initially, it should be mentioned that a larger ϵ generates two opposite effects on the average productivity of the qualified sector. According to Proposition 1, an increase in this term leads to a reduction in Q_d and an increase in Q_o. Thus, if the first effect dominates the second one, there is a decrease in the average productivity of the skilled sector. Otherwise, we have the opposite effect. The last proposition presents a condition that ensures that the skilled sector productivity grows with ϵ.

We present, in Figure 4, two cases in which there is and there is not an average productivity gain with the school supply policy. In the first figure, the situation in which the distribution of school quality has a long tail to the left is presented. In this case, the educational policy brings a positive gain for average skill sector productivity, as mentioned in proposition 3. In the figure below, we present another possible situation, when the mass of the distribution is more concentrated to the left. In this case, the policy will be followed by a drop in the average quality of the schooled sector.

Now, to better understand the impact of an open enrollment policy on the labor market, consider that the condition of the previous proposition is fulfilled[30]. Thus, the aggregate productivity of the skilled sector increases. The higher aggregate productivity in the skilled sector also generates an increase in the high-quality job creation flow. However, since there is also an increase in the measure of skilled workers in the economy, the impact of a higher ϵ on the skilled sector unemployment rate is uncertain. If the job creation dynamics in the skilled sector dominate the increase in the mass of skilled workers, there is a reduction in unemployment. Otherwise, there will be an increase at the unemployment rate. The skilled sector wage rate also increases. This happens due to higher productivity in the skilled sector and the greater job creation dynamics in this segment.

Now we consider the unskilled sector. Since a larger ϵ only brings a reduction of the measure of unskilled workers in our economy, there is a drop of the unemployment rate in this sector. The wage rate and the job creation dynamics in this sector remain unchanged. Therefore, in sum, an open enrollment policy generates a higher job creation dynamics, a higher wage rate and a lower unemployment rate in the skilled sector if the job creation effect dominates the increase in the skilled labor force. The impact on the unqualified sector is only indirect and occurs through a

[30]This case is presented at the top of Figure 4 (when $[Q_o - q^e(Q_d, Q_o)]g(Q_o) > [q^e(Q_d, Q_o) - Q_d]g(Q_d)$).

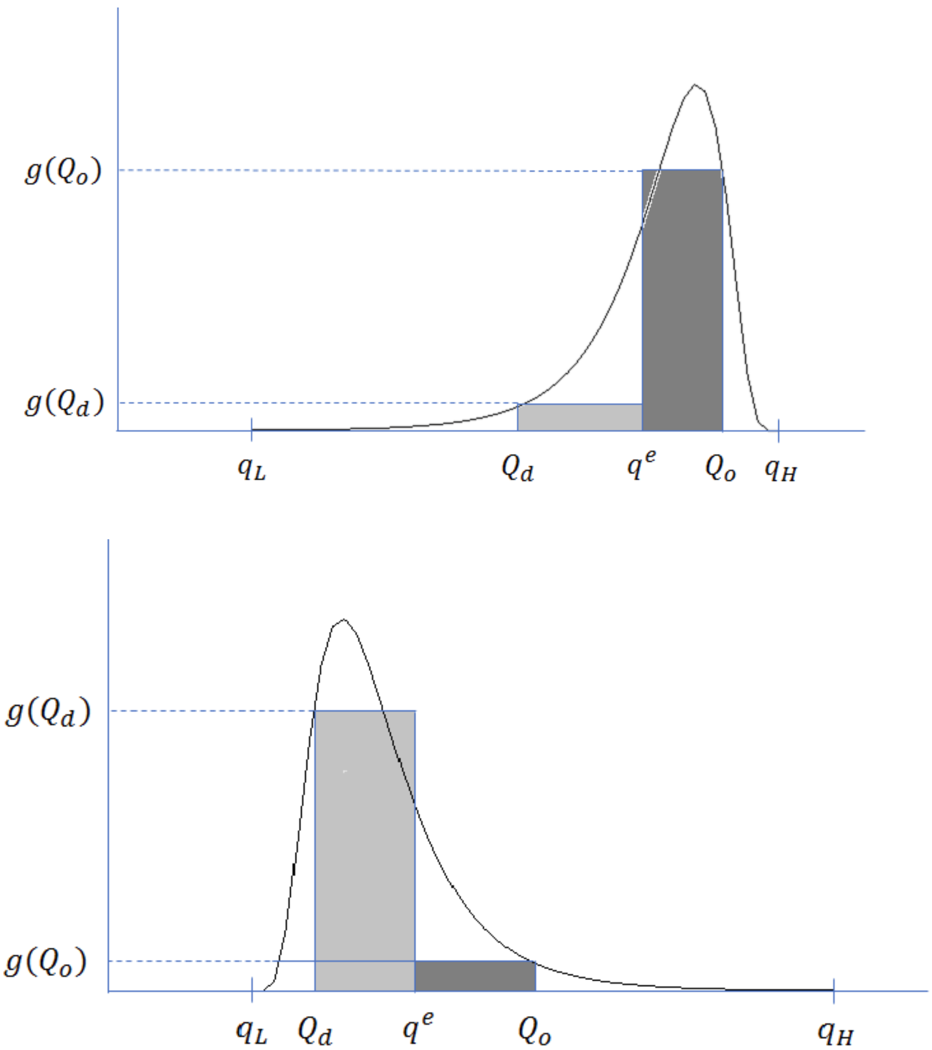

Figure 3: School Quality Distribution

reduction in the measure of uneducated individuals in the economy, thus reducing the unemployment rate in this sector.

Now consider the other educational policy distortion, that is, the random assignment of school places to students. Consider that government policy is now given by a degenerate $\bar{q}$-quality school vacancy provided to all individuals in our economy, rather than a school-quality lottery in the set $[Q_d, Q_o]$. The following proposition guarantees there is degenerated school quality provision - strictly lower than q_H - that generates an increase in the average productivity of the skilled sector and so an increase in the job creation dynamics in the skilled sector.

Proposition 4 *Let $\bar{I} \in [I_L, I_H]$ such that $\bar{q} = Q_o(\bar{I})$ and $\bar{q}$ represents the linear school quality provided by the government to all agents. Then, there exists an $\delta \in [I_L, \bar{I}]$ such that if $\bar{I} < \delta$ we have:*

$$\bar{q} > q^e(Q_d, Q_o).$$

Proof. See Appendix C. ■

For a better understanding of the impacts of this policy, consider Figure 5. As mentioned in the previous section, $\tilde{I}$ characterizes the threshold education cost that defines the measure of skilled and unskilled workers in the economy when the government implements $Q_o(I)$, defined by expression (23), and randomly assigns students to school places in the interval $[Q_d(I), Q_o(I)]$. In equilibrium, the productivities of the skilled and unskilled segments of the economy are given by $q^e(Q_d, Q_o)$ and q_L, respectively. Agents with individual cost of education defined in the subset $[\tilde{I}, I_H]$ never study. Also, the group of individuals with individual cost of education defined in the subset $[I_L, \tilde{I})$ and with school quality provision are outside the gray area in Figure 5.

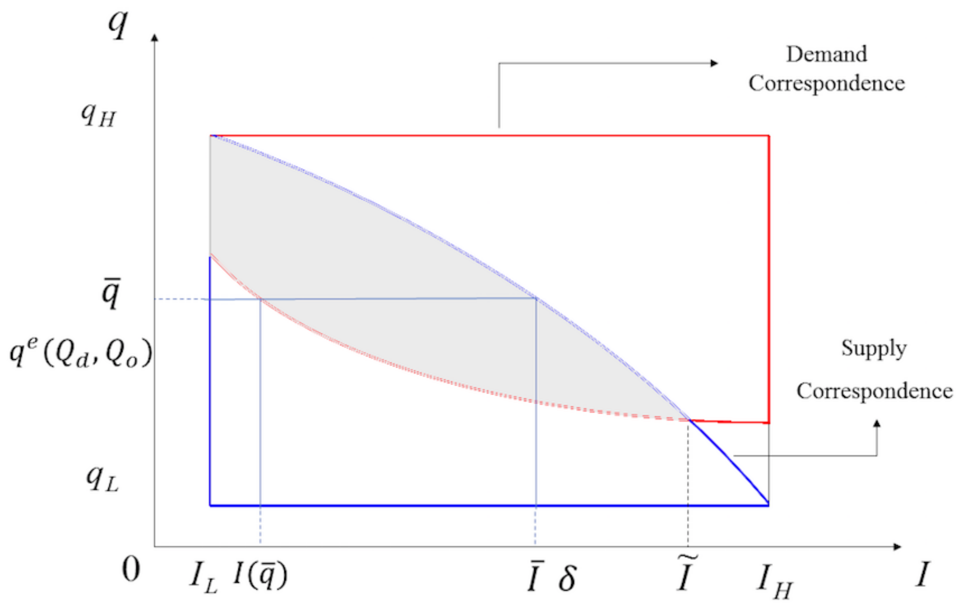

Figure 4: Linear School Quality Provision.

Now consider the policy that offers $\bar{q}$ to all individuals. The previous proposition guarantees that there exists an $\bar{I} < \delta$ such that $\bar{q} > q^e(Q_d, Q_o)$, i.e., there is a linear school quality $\bar{q}$ that once supplied to all agents generates an increase in the skilled sector productivity. This higher productivity in turn brings an increase in both the job creation dynamics and the wage rate in the skilled sector. The impact on the unskilled sector is again indirect and emerges through a reduction on the measure of non-educated individuals in our economy.

Note that the labor market outcomes generated by this policy resemble those previously discussed. The main difference between these policies lies in the fact that, instead of excluding individuals with higher education costs, there is now removal of individuals with the lowest cost of education from the skilled segment of the economy, which is a sign that this policy is followed by welfare losses[31].

It should also be mentioned that although such a policy looks attractive by ensuring equal opportunities in students' access to education, it can be detrimental to the economy. For instance, if this singular school quality provision is very low and given by $\bar{q} < q^e(Q_d, Q_o)$, there is both a fall in workers' productivity and a reduction in the job creation dynamics in the skilled sector. Moreover, whenever the mass of uneducated agents increases, there is also an increase in the unskilled unemployment rate.

4 Centralized Equilibrium and Inefficiency

It is a well-known fact that labor market imperfections generate inefficient labor market outcomes. The link that exists among labor market tightness and worker and firm transition probabilities implies that bargained wages do not fully internalize the search externality, unless the Hosios Condition is present[32]. This happens basically because once matched, firms and workers do not consider the effects of their decisions on the agents still searching for a productive partner. The consequence is that the equilibrium outcome is socially inefficient.

We saw in the previous section that with heterogeneous schooling costs, a degenerate school quality supply policy removes students with the lowest schooling costs from the skilled sector, which seems to be a costly and inefficient policy option. However, this result does not guarantee that the random assignment of students is an efficient policy. This section has two main goals. The first is to characterize efficient allocations in an economy characterized by open enrollment and random assignment of students to school places. And second, we evaluate whether inefficiency prevails in the decentralized equilibrium, whenever there is open enrollment and, so once there is oversubscription, the students' selection is made by admission lotteries.

[31] Notice that the measure of schooled agents in the economy is now defined by $1 - H(I(\bar{q})) = \int_{I(\bar{q})}^{I_H} dH(I)$ instead of $H(\tilde{I}) = \int_{I_L}^{\tilde{I}} dH(I)$ with the random supply policy.

[32] This condition states that if the firms' bargaining power equalizes the elasticity of the matching function, the decentralized equilibrium is efficient with regards to vacancies. See Pissarides (2000) and Hosios (1990) for more on this.

Therefore, the social planner solves the following problem:

$$\max_{Q,v_S,v_N,l_S,l_N} \mathcal{W}(Q,v_S,v_N,l_S,l_N) = \int_0^\infty e^{-\rho t}\rho[q_L l_N^{\alpha_N} + u_N b_N - k_N v_N - C_N]dt$$

$$+\int_T^\infty e^{-\rho(t-T)}\rho[q^e(Q) l_S^{\alpha_S} + u_S b_S - k_S v_S - C_S]dt - [1-G(Q)]\int_0^T \int_{I_L}^{I_H} e^{-\rho t}\rho I Q_d(I) dH(I) dt;$$

subject to:

$$p(\theta_i)v_i = \lambda_i l_i \qquad \text{and} \qquad \theta_i = \tfrac{v_i}{u_i}, \qquad \text{for } i=\{S,N\};$$

$$u_N = G(Q) - l_N \qquad \text{and} \qquad u_S = 1 - G(Q) - l_S;$$

$$Q_d(I) = \frac{(b_S + \frac{\beta_S}{1-\beta_S}k_s\theta_S) - (b_N + \frac{\beta_N}{1-\beta_N}k_n\theta_N)}{\rho(1-e^{-\rho T})I}.$$

Notice that the first term on the right-hand side of the objective function corresponds to the output and the benefits enjoyed by employed and unemployed workers in the unskilled sector. This amount is deduced by the cost of opening a new vacancy in the unskilled sector, C_N. The following term is identical to the first one, but it refers to the skilled sector. The final term is related to schooling costs. Notice that the total cost of education is composed of the compulsory period of schooling (T), the individual cost of education (I), and the individual demand for school quality ($Q_d(I)$). The previous problem can be restated as[33]:

$$\max_{Q,v_S,v_N,l_S,l_N} \mathcal{W}(Q,v_S,v_N,l_S,l_N) = [q_L l_N^{\alpha_N} + u_N b_N - k_N v_N - C_N] + [q^e(Q) l_S^{\alpha_S} + u_S b_S - k_S v_S - C_S] - [1-G(Q)]\frac{1}{\rho}[(b_S + \frac{\beta_S}{1-\beta_S}k_s\theta_S) - (b_N + \frac{\beta_N}{1-\beta_N}k_n\theta_N)]; \tag{26}$$

subject to:

$$p(\theta_i)v_i = \lambda_i l_i \qquad \text{and} \qquad \theta_i = \tfrac{v_i}{u_i}, \qquad \text{for } i=\{S,N\};$$

$$u_N = G(Q) - l_N \qquad \text{and} \qquad u_S = 1 - G(Q) - l_S.$$

It is worth noting that we have adopted a strategy different from the traditional one used in the literature. Due to the difficulty of obtaining comparable expressions for the centralized and the distortionary decentralized equilibrium we were forced to impose some additional conditions in this section. Namely, we study the impact of a

[33] Please refer to Appendix D for the solution of the central planner problem.

random assignment of school places to students after eliminating the first distortion, and both the congestion and the large firms' externalities[34]. We then show that even by eliminating these externalities, we still cannot restore the efficient outcome[35].

Let $(\theta_N^P, \theta_S^P, Q^P)$ be the solution of the centralized problem. The following proposition suggests a parallel between the social planner and the decentralized equilibrium allocations. It shows there is excessive job creation in the unskilled sector and reduced job creation in the skilled sector whenever $Q_d < Q^P$.

Proposition 5 *Let $(\theta_N, \theta_S, Q_d, Q_o)$ represent the equilibrium allocations in the decentralized economy. Consider that: $\alpha_N = \alpha_S = 1$; $C_N = C_S = \frac{1}{(1+\rho)}$; the Hosios Condition is satisfied; and ρ is small enough. Then:*

(i) There is overemployment in the unskilled sector ($\theta_N > \theta_N^P$);

(ii) There is underemployment in the skilled sector ($\theta_S < \theta_S^P$) if $Q^P > Q_d$.

Proof. See Appendix E. ■

The previous proposition shows that even after an open enrollment policy is imposed, the decentralized economy is still inefficient. There is overemployment in the unskilled sector. In turn, we can obtain either over or underemployment in the skilled sector. The previous proposition also ensures that whenever $Q^P > Q_d$, there is excess of skilled workers and a low average productivity of the skilled sector at the decentralized equilibrium, when compared to the efficient result. As a consequence, there is excess of job creation in the unskilled sector (overemployment) and low creation of employment in the skilled segment (underemployment) when compared to the centralized equilibrium.

To better understand this result, consider Figure 6. In the top figure, we introduce the scenario where average productivity in the decentralized economy is lower than the efficient productivity, $q^e(Q_d, Q_s) < q^e(Q^P, q_H)$, whereas in the other figure, we have that $q^e(Q_d, Q_s) > q^e(Q^P, q_H)$. Notice, in the first figure, that the lower average productivity at the decentralized equilibrium implies there is an excess of skilled workers in the decentralized scenario. Thus, an admission lottery policy per se does not drive the economy to an efficient outcome.

[34]Our strategy in this section is to impose some efficiency conditions and check if the equilibrium remains efficient. Thus, in addition to imposing the Hosios Condition, we also assume $\epsilon \to \infty$ and constant returns to scale to eliminate the large firms' externality. See Cahuc and Wasmer (2001) and Smith (1999) on this point.

[35]In sum, the problem of the social planner consists of defining the measure of individuals who study, $[Q^P, q_H]$. Then, the school to be attended by each individual is, as before, randomly defined

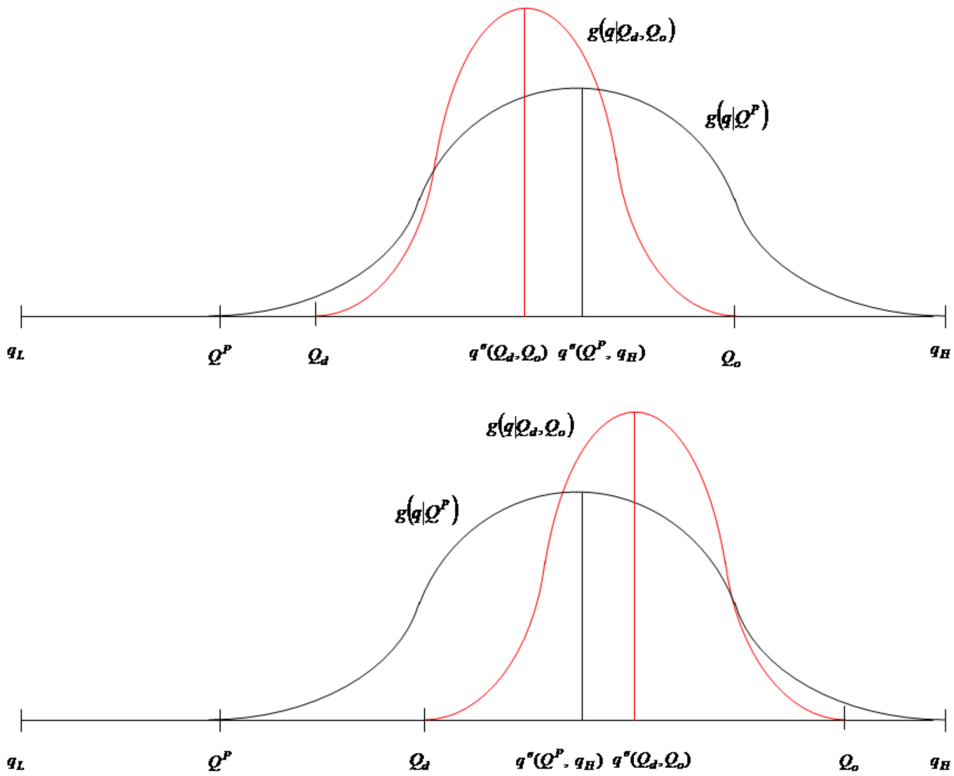

Figure 5: Average Productivity

A final interesting point to be observed from these two figures is that even though there is excessive demand for education ($Q_d < Q^P$), the education supply policy may be effective in restoring the efficient average quality of the skilled labor force, $q^e(Q_d, Q_s) = q^e(Q^P, q_H)$, thus eliminating the impact of an inefficient amount of schooling investments. For instance, the government may resort to changes in the school quality distribution as a mechanism to restore the efficient outcomes in the skilled and the unskilled segments of the economy.

5 Concluding Remarks

Policies that promote the skilled sector have become widespread. Based on the evidence that governments can affect the size and the quality of the educated labor force to make certain regions more attractive, many countries have spent significant

in this subset. The social planner also defines v_i and l_i, for $i = \{S, N\}$. Notice that the expected productivity in this subset is now given by $q^e(Q^P, q_H) = E[q \mid q \geq Q^P]$. In Appendix D we present the detailed solution to this problem.

amounts of cash to boost school enrollment and increase equity and efficiency in individual access to education.

The main objective of this paper is to study the effects of an education policy reform that seeks to reduce distortions in the public supply of education. In particular, we are interested in evaluating the impact of both an open enrollment policy and one that seeks equity in the provision of education in the labor market.

We show that in a general equilibrium model with labor market frictions, that both open enrollment and school equity policies have ambiguous effects on the labor market. Whenever their impact on the number of workers choosing to become educated are stronger than the additional school quality gains generated by the policy, the effects on the economy are negative. We also demonstrate that an admission lottery policy generates inefficient economic outcomes.

Appendix

Appendix A

For each ϵ, let $\tilde{I}(\epsilon)$ be the solution of $Q_o(\tilde{I}(\epsilon)) = Q_d(\tilde{I}(\epsilon))$. Now, consider an increase from ϵ to ϵ' and let:

$$Q_o'(I) = q_H - (q_H - q_L)\left(\frac{I - I_L}{I_H - I_L}\right)^{\epsilon'}.$$

It follows that $Q_o'(I) > Q_o(I)$, for all $I \in (I_L, I_H)$. Since $\tilde{I} \in (I_L, I_H)$ for any $\epsilon \geq 1$we have that:

$$Q_o'(\tilde{I}(\epsilon)) > Q_d(\tilde{I}(\epsilon)).$$

However, from the definition of $\tilde{I}(\epsilon')$, we have that:

$$Q_o'(\tilde{I}(\epsilon')) = Q_d(\tilde{I}(\epsilon')).$$

Adding the last two expressions and multiplying by $1/2$ yields:

$$\frac{1}{2}Q_o'(\tilde{I}(\epsilon)) + \frac{1}{2}Q_o'(\tilde{I}(\epsilon')) > \frac{1}{2}Q_d(\tilde{I}(\epsilon)) + \frac{1}{2}Q_d(\tilde{I}(\epsilon')).$$

By the concavity of Q_o' and the strict convexity of Q_d, it follows that:

$$Q_o'\left(\frac{\tilde{I}(\epsilon) + \tilde{I}(\epsilon')}{2}\right) > Q_d\left(\frac{\tilde{I}(\epsilon) + \tilde{I}(\epsilon')}{2}\right).$$

From these two last expressions, we have that:

$$\frac{\tilde{I}(\epsilon) + \tilde{I}(\epsilon')}{2} < \tilde{I}(\epsilon')$$

then, $\tilde{I}(\epsilon) < \tilde{I}(\epsilon')$.

Now, we show that Q_d is decreasing in $\tilde{I}$. Consider that:

$$Q_d = M\nu_d,$$

for:

$$M = \frac{\frac{1}{\rho}\left[\left(b_S + \frac{\beta_S}{1-\beta_S}k_S\theta_S\right) - \left(b_N + \frac{\beta_N}{1-\beta_N}k_N\theta_N\right)\right]}{(1 - e^{-\rho T})};$$

and

$$\nu_d = \mathbb{E}\left[\frac{1}{I} \mid I \leq \tilde{I}\right].$$

Notice that changes in Q_d, due to changes in $\tilde{I}$, occur through the term ν_d. We may rewrite ν_d as:

$$\nu_d = \mathbb{E}\left[x | x \geq \tilde{x}\right],$$

where $x = {}^1/_I$ and $\tilde{x} = {}^1/_{\tilde{I}}$.

Then, increases in $\tilde{I}$ are equivalent to reductions of $\tilde{x}$ and ν_d. This guarantees that the higher ϵ is, the lower Q_d will be.

Now we verify the impact of ϵ on Q_o. Consider that:

$$Q_o = q_H - (q_H - q_L)\nu_o$$

for

$$\nu_o = \mathbb{E}\left[\left(\frac{I - I_L}{I_H - I_L}\right)^{\epsilon} | I \leq \tilde{I}\right].$$

As previously, consider that:

$$x = \frac{I - I_L}{I_H - I_L} \in [0, 1]$$

and

$$\tilde{x} = \frac{\tilde{I} - I_L}{I_H - I_L} \in [0, 1]$$

for all $I \in [I_L, I_H]$. Then,

$$\nu_o = \int_0^{\tilde{x}} x^{\epsilon} \frac{d\tilde{H}(x)}{\tilde{H}(\tilde{x})},$$

where $\tilde{H}(x)$ is the distribution of transformation $x = (I - I_L)/(I_H - I_L)$. Differentiating this last expression with respect to ϵ, we get

$$\frac{d\nu_o}{d\epsilon} = \left[(\tilde{x}^{\epsilon} - \nu_o)\tilde{h}(\tilde{x}) + \int_0^{\tilde{x}} x^{\epsilon} ln(x) \frac{d\tilde{H}(x)}{\tilde{H}(\tilde{x})}\right] \frac{1}{\tilde{H}(\tilde{x})} \frac{d\tilde{x}}{d\epsilon} < 0.$$

The sign of the previous expression is derived from: $\tilde{x}^{\epsilon} - \nu_o < 0$, $0 < x < 1$ and $d\tilde{x}/d\epsilon \propto d\tilde{I}/d\epsilon$.

Appendix B

Proof. To simplify the proof, consider that $q^e(Q_d, Q_o) = q^e$. We have that:

$$\frac{dq^e}{d\varepsilon} = \frac{\partial q^e}{\partial Q_d} \frac{dQ_d}{d\varepsilon} + \frac{\partial q^e}{\partial Q_o} \frac{dQ_o}{d\varepsilon}.$$

By using the fact that $\frac{\partial q^e(Q_d,Q_o)}{\partial Q_d} > 0$, $\frac{\partial q^e(Q_d,Q_o)}{\partial Q_o} > 0$ and Proposition 1, we have that all previous derivatives are positive except $dQ_d/d\varepsilon$.

We now rewrite the previous expression as:

$$\frac{dq^e}{d\varepsilon} = -\frac{[q^e - Q_d]g(Q_d)}{G(Q_o) - G(Q_d)} \left|\frac{dQ_d}{d\varepsilon}\right| + \frac{[Q_o - q^e]g(Q_o)}{G(Q_o) - G(Q_d)} \frac{dQ_o}{d\varepsilon}.$$

Therefore, we have that $dq^e/d\varepsilon > 0$ if and only if:

$$\frac{[Q_o - q^e]g(Q_o)}{[q^e - Q_d]g(Q_d)} > \frac{\left|\frac{dQ_d}{d\varepsilon}\right|}{\frac{dQ_o}{d\varepsilon}}.$$

To complete the demonstration we need the following result.

Lemma For any values of Q_d and Q_o in $[q_L, q_H]$ and $\varepsilon \in [1, \infty)$, we have:

$$\left|\frac{dQ_d}{d\varepsilon}\right| < \frac{dQ_o}{d\varepsilon}.$$

Proof. By definition we have:

$$Q_d = \frac{U_S - U_N}{(1 - e^{-\rho T})} \int_{I_L}^{I(\varepsilon)} \frac{1}{s} \frac{h(s)}{H(I(\varepsilon))} ds$$

where $I(\varepsilon)$ is such that $Q_d(I(\varepsilon)) = Q_o(I(\varepsilon))$. Differentiating the previous expression with respect to ε yields:

$$\begin{aligned}
\frac{dQ_d}{d\varepsilon} &= \frac{U_S - U_N}{(1 - e^{-\rho T})} \left[\frac{1}{I(\varepsilon)} \frac{h(I(\varepsilon))}{H(I(\varepsilon))} \frac{dI}{d\varepsilon} - \int_{I_L}^{I(\varepsilon)} \frac{1}{s} \frac{h(s)}{H(I(\varepsilon))} \frac{h(I(\varepsilon))}{H(I(\varepsilon))} \frac{dI}{d\varepsilon} ds \right] \\
&= \frac{U_S - U_N}{(1 - e^{-\rho T})} \left[\frac{1}{I(\varepsilon)} - \int_{I_L}^{I(\varepsilon)} \frac{1}{s} \frac{h(s)}{H(I(\varepsilon))} ds \right] \frac{h(I(\varepsilon))}{H(I(\varepsilon))} \frac{dI}{d\varepsilon} \\
&= \left[Q_d(I(\varepsilon)) - Q_d \right] \frac{h(I(\varepsilon))}{H(I(\varepsilon))} \frac{dI}{d\varepsilon}.
\end{aligned}$$

It is easy to shown that for any $\varepsilon \in [1, \infty)$, $Q_d(I(\varepsilon)) < Q_d$. In addition, from the proof of Proposition 1 in Appendix A, it is shown that $dI/d\varepsilon > 0$. Thus we have:

$$\left|\frac{dQ_d}{d\varepsilon}\right| = \left[Q_d - Q_d(I(\varepsilon)) \right] \frac{h(I(\varepsilon))}{H(I(\varepsilon))} \frac{dI}{d\varepsilon}. \tag{27}$$

By using the definition of Q_o, we have:

$$Q_o = q_H - (q_H - q_L) \int_{I_L}^{I(\varepsilon)} \left(\frac{s - I_L}{I_H - I_L} \right)^{\varepsilon} \frac{h(s)}{H(I(\varepsilon))} ds.$$

Differentiating this previous expression with respect to ε gives us:

$$
\begin{aligned}
\frac{dQ_o}{d\varepsilon} &= -(q_H - q_L)\left\{\left(\frac{I(\varepsilon)-I_L}{I_H-I_L}\right)^{\varepsilon}\frac{h(I(\varepsilon))}{H(I(\varepsilon))}\frac{dI}{d\varepsilon}\right. \\
&\quad + \int_{I_L}^{I(\varepsilon)}\left(\frac{s-I_L}{I_H-I_L}\right)^{\varepsilon}\ln\left(\frac{s-I_L}{I_H-I_L}\right)\frac{h(s)}{H(I(\varepsilon))}ds \\
&\quad \left. - \int_{I_L}^{I(\varepsilon)}\left(\frac{s-I_L}{I_H-I_L}\right)^{\varepsilon}\frac{h(s)}{H(I(\varepsilon))}\frac{h(I(\varepsilon))}{H(I(\varepsilon))}ds\right\}. \qquad (28)
\end{aligned}
$$

Manipulating the previous expression:

$$
\begin{aligned}
\frac{dQ_o}{d\varepsilon} &= \left[-(q_H - q_L)\left(\frac{I(\varepsilon)-I_L}{I_H-I_L}\right)^{\varepsilon} + \right. \\
&\quad \left.(q_H - q_L)\int_{I_L}^{I(\varepsilon)}\left(\frac{s-I_L}{I_H-I_L}\right)^{\varepsilon}\frac{h(s)}{H(I(\varepsilon))}ds\right]\frac{h(I(\varepsilon))}{H(I(\varepsilon))}\frac{dI}{d\varepsilon} \\
&\quad -(q_H - q_L)\int_{I_L}^{I(\varepsilon)}\left(\frac{s-I_L}{I_H-I_L}\right)^{\varepsilon}\ln\left(\frac{s-I_L}{I_H-I_L}\right)\frac{h(s)}{H(I(\varepsilon))}ds. \qquad (29)
\end{aligned}
$$

By Adding and subtracting q_H in the term between brackets, we have:

$$
\frac{dQ_o}{d\varepsilon} = [Q_o(I(\varepsilon)) - Q_o]\frac{h(I(\varepsilon))}{H(I(\varepsilon))}\frac{dI}{d\varepsilon} - (q_H - q_L)\int_{I_L}^{I(\varepsilon)}\left(\frac{s-I_L}{I_H-I_L}\right)^{\varepsilon}\ln\left(\frac{s-I_L}{I_H-I_L}\right)\frac{h(s)}{H(I(\varepsilon))}ds.
$$

Note that the above expression is positive, since $Q_o(I(\varepsilon)) > Q_o$ for any $\varepsilon \in [1, \infty)$ and $\ln[(s-I_L)/(I_H-I_L)] < 0$ for any $s < I_H$. Finally, using (27) and (29), we have:

$$\left|\frac{dQ_d}{d\varepsilon}\right| - \frac{dQ_o}{d\varepsilon} = [Q_d - Q_d(I(\varepsilon))]\frac{h(I(\varepsilon))}{H(I(\varepsilon))}\frac{dI}{d\varepsilon}$$

$$-[Q_o(I(\varepsilon)) - Q_o]\frac{h(I(\varepsilon))}{H(I(\varepsilon))}\frac{dI}{d\varepsilon}$$

$$+(q_H - q_L)\int_{I_L}^{I(\varepsilon)}\left(\frac{s-I_L}{I_H-I_L}\right)^{\varepsilon}\ln\left(\frac{s-I_L}{I_H-I_L}\right)\frac{h(s)}{H(I(\varepsilon))}ds$$

$$= [(Q_o + Q_d) - (Q_o(I(\varepsilon)) + Q_d(I(\varepsilon)))]\frac{h(I(\varepsilon))}{H(I(\varepsilon))}\frac{dI}{d\varepsilon}$$

$$+(q_H - q_L)\int_{I_L}^{I(\varepsilon)}\left(\frac{s-I_L}{I_H-I_L}\right)^{\varepsilon}\ln\left(\frac{s-I_L}{I_H-I_L}\right)\frac{h(s)}{H(I(\varepsilon))}ds.$$

By definition $Q_o(I(\varepsilon)) = Q_d(I(\varepsilon))$ such that $Q_o(I(\varepsilon)) + Q_d(I(\varepsilon)) = 2Q_o(I(\varepsilon))$. Thus, we have $Q_o(I(\varepsilon)) + Q_d(I(\varepsilon)) = 2Q_o(I(\varepsilon)) > 2Q_o > Q_o + Q_d$. Therefore:

$$\left|\frac{dQ_d}{d\varepsilon}\right| < \frac{dQ_o}{d\varepsilon}.$$

■

Using the previous result in the proposition, we have that:

$$\frac{[Q_o - q^e]g(Q_o)}{[q^e - Q_d]g(Q_d)} > 1 > \frac{\left|\frac{dQ_d}{d\varepsilon}\right|}{\frac{dQ_o}{d\varepsilon}}$$

or equivalently:

$$\frac{d}{d\varepsilon}q^e(Q_d, Q_o) > 0.$$

■

Appendix C

Initially, it should be mentioned that:

$$Q_o \geq q^e(Q_d, Q_o)$$

Thus, it suffices to show that $\bar{q} > Q_o$. By definition, we have that:

$$Q_o = q_H - (q_H - q_L)\int_{I_L}^{\tilde{I}}\left(\frac{I-I_L}{I_H-I_L}\right)^{\varepsilon}\frac{h(I)}{H(\tilde{I})}dI.$$

As $h(I) \geq 0$ for any $I \in [I_L, \tilde{I}]$, the mean value theorem for integrals guarantees that there exists $\delta \in [I_L, \tilde{I}]$ such that:

$$\int_{I_L}^{\tilde{I}} \left(\frac{I - I_L}{I_H - I_L}\right)^{\varepsilon} \frac{h(I)}{H(\tilde{I})} dI = \left(\frac{\delta - I_L}{I_H - I_L}\right)^{\varepsilon} \int_{I_L}^{\tilde{I}} \frac{h(I)}{H(\tilde{I})} dI = \left(\frac{\delta - I_L}{I_H - I_L}\right)^{\varepsilon}.$$

Therefore, if $\bar{I} < \delta$ then:

$$\left(\frac{\bar{I} - I_L}{I_H - I_L}\right)^{\varepsilon} < \left(\frac{\delta - I_L}{I_H - I_L}\right)^{\varepsilon} = \int_{I_L}^{\tilde{I}} \left(\frac{I - I_L}{I_H - I_L}\right)^{\varepsilon} \frac{h(I)}{H(\tilde{I})} dI$$

which implies the desired result.

■

Appendix D

We initially solve the centralized problem. Consider again the problem presented in (26). Manipulating the previous constraints, the central planner problem can be redefined as:

$$\max_{Q, \theta_S, \theta_N} \mathcal{W}(Q, v_S, v_N, l_S, l_N) = G(Q) \left\{ q_L [\frac{z(\theta_N) G(Q)}{\lambda_N + z(\theta_N)}]^{\alpha_N} \frac{1}{G(Q)} + \frac{\lambda_N b_N - k_N \lambda_N \theta_N}{\lambda_N + z(\theta_N)} - \frac{C_N}{G(Q)} \right\} +$$
$$[1 - G(Q)] \left\{ q^e(Q) [\frac{z(\theta_S)[1 - G(Q)]}{\lambda_S + z(\theta_S)}]^{\alpha_S} \frac{1}{[1 - G(Q)]} + \frac{\lambda_S b_S - k_S \lambda_S \theta_S}{\lambda_S + z(\theta_S)} - \frac{C_S}{[1 - G(Q)]} \right\} -$$
$$[1 - G(Q)] \frac{1}{\rho} [(b_S + \frac{\beta_S}{1 - \beta_S} k_S \theta_S) - (b_N + \frac{\beta_N}{1 - \beta_N} k_N \theta_N)].$$

Assume that the Hosios Condition is met[36]. The set of expressions that characterize the social optimum is given by:

$$\frac{k_N(\lambda_N + \beta_N z(\theta_N^P))}{p(\theta_N^P))} = \alpha_N q_L (1 - \beta_N) [\frac{z(\theta_N^P) G(Q^P)}{\lambda_N + z(\theta_N^P)}]^{(\alpha_N - 1)} \tag{30}$$
$$- \frac{1}{\rho} [\frac{1 - G(Q^P)}{G(Q^P)}] \frac{\beta_N k_N}{(1 - \beta_N) p(\theta_N^P)} - (1 - \beta_N) b_N;$$

[36]The Hosios Condition states that $\beta_i = 1 - \eta_i$, for $i = \{S, N\}$ and $\eta_i = \frac{z'(\theta_i)}{z(\theta_i)} \theta_i$ represents the elasticity of the job matching with regards to a job vacancy. Notice that by imposing the Hosios Condition in the social planner problem we expunge the trade externality of our model.

$$\frac{k_S(\lambda_S+\beta_S z(\theta_S^P))}{p(\theta_S^P))} = \alpha_S q^e(Q^P)(1-\beta_S)[\frac{z(\theta_S^P)(1-G(Q^P))}{\lambda_S+z(\theta_S^P)}]^{(\alpha_S-1)} \\ +\frac{1}{\rho}[\frac{1-G(Q^P)}{G(Q^P)}]\frac{\beta_S k_S}{(1-\beta_S)p(\theta_S^P)} - (1-\beta_S)b_S; \tag{31}$$

$$\mathcal{A}(\theta_N^P)G(Q^P)^{\alpha_N-1}q_L + [1-G(Q^P)]^{\alpha_S-1}[\mathcal{B}(\theta_S^P)q^e(Q^P) - \mathcal{C}(\theta_S^P)Q^P] \\ = \mathcal{D}(\theta_N^P) - \mathcal{E}(\theta_S^P) - \mathcal{F}(\theta_N^P,\theta_S^P); \tag{32}$$

where:

$$\mathcal{A}(\theta_N^P) = \alpha_N[\frac{z(\theta_N^P)}{\lambda_N+z(\theta_N^P)}]^{\alpha_N}; \quad \mathcal{B}(\theta_S^P) = (1-\alpha_S)[\frac{z(\theta_S^P)}{\lambda_S+z(\theta_S^P))}]^{\alpha_S};$$

$$\mathcal{C}(\theta_S^P) = \alpha_S[\frac{z(\theta_S^P)}{\lambda_S+z(\theta_S^P))}]^{\alpha_S}; \quad \mathcal{D}(\theta_N^P) = \frac{k_N\lambda_N\theta_N^P - \lambda_N b_N}{\lambda_N+z(\theta_N^P)};$$

$$\mathcal{E}(\theta_S^P) = \frac{k_S\lambda_S\theta_S^P - \lambda_S b_S}{\lambda_S+z(\theta_S^P)};$$

$$\mathcal{F}(\theta_N^P,\theta_S^P) = \frac{1}{\rho}[(b_S+\frac{\beta_S}{1-\beta_S}k_S\theta_S^P) - (b_N+\frac{\beta_N}{1-\beta_N}k_N\theta_N^P)];$$

and θ_S^P, θ_N^P and Q^P represent the efficient values of the market tightness in the skilled and unskilled sectors and the efficient mass of skilled individuals, respectively.

Notice from (30) that the cost of opening a new vacancy in the unskilled sector equalizes the social surplus generated by this sector. This expression defines the optimal steady-state value of θ_N^P. The following equation characterizes the efficient value of θ_S^P. Finally, expression (32) determines the optimal mass of the educated workforce that generates the efficient outcome.

Appendix E

Let

$$H_N(\theta;\rho) = \frac{(\rho+\lambda_N+\beta_N q(\theta))k_N}{p(\theta)};$$

$$H_S(\theta;\rho) = \frac{(\rho+\lambda_S+\beta_S q(\theta))k_S}{p(\theta)};$$

$$H_N^P(\theta) = \frac{(\lambda_N + \beta_N q(\theta))k_N}{p(\theta)};$$

$$H_S^P(\theta) = \frac{(\lambda_S + \beta_S q(\theta))k_S}{p(\theta)}.$$

It can be seen that all previous expressions are increasing in θ. Furthermore, note that for any θ:

$$H_j^P(\theta) = \lim_{\rho \to 0} H_j(\theta; \rho), \qquad for \quad j = N, S. \tag{33}$$

Consider that $\alpha_N = \alpha_S = 1$, $C_N = C_S = \frac{1}{(1+\rho)}$ and the Hosios Condition are satisfied. In this case, the equilibrium expressions of θ_N and θ_N^P satisfy, respectively:

$$H_N(\theta_N; \rho) = (1 - \beta_N)(q_L - b_N); \tag{34}$$

$$H_N^P(\theta_N^P) = (1 - \beta_N)(q_L - b_N) - [\frac{1 - G(Q^P)}{\rho G(Q^P)}][\frac{\beta_N k_N}{(1 - \beta_N)p(\theta_N)}]. \tag{35}$$

By using (34) and (35) we have that:

$$H_N^P(\theta_N^P) < H_N(\theta_N; \rho).$$

At the limit, when $\rho \to 0$ and using (33), we conclude that:

$$H_N^P(\theta_N^P) < H_N^P(\theta_N).$$

Then $\theta_N^P < \theta_N$.

By using the same reasoning for the expressions that define θ_S and θ_S^P we have that:

$$H_S^P(\theta_S^P) > H_S(\theta_S; \rho) + (1 - \beta_S)[q^e(Q^P) - q^e(Q_d, Q^o)]. \tag{36}$$

By definition, $q^e(Q^P) = q^e(Q^P, q_H)$. Thus:

$$q^e(Q^P) \geq q^e(Q^P, Q_o), \quad for\ any \quad Q_o \in [q_L, q_H].$$

Therefore, if $Q^P > Q_d$ then $q^e(Q^P) > q^e(Q_d, Q^o)$. By using (36) we have that:

$$H_S^P(\theta_S^P) > H_S(\theta_S; \rho).$$

Finally, taking the limit when $\rho \to 0$, we have:

$$H_S^P(\theta_S^P) > H_S^P(\theta_S).$$

Then $\theta_N^P < \theta_N$.

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Is it always worth implementing an open enrollment policy? And implementing policies that pursue equity in school supply? What is the impact of these two policies on the labor market? Do they produce efficient outcomes? This paper theoretically provides answers to these questions by studying the link between distortionary school supply policies and labor market performance. We build a two-sector labor market matching model, where the skilled segment of the economy is composed of workers who differ in the quality of the school they attended. We show the impact of government interventions to eliminate educational supply policy distortions within this theory. We demonstrate that both open enrollment and school equity policies have ambiguous effects on the labor market. Whenever their impact on the measure of workers choosing to become better educated is stronger than the additional school quality gains generated by the policy, the effects on the economy are negative. We also study the central planner solution, emphasizing the existing inefficiencies.

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