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ABSTRACT

I incorporate household debt and delinquency decisions into a standard model of lifecycle consumption-saving-investment. Deriving this extended system of Euler equations does not alter consumption-based fundamental asset pricing equation. It imposes that the pricing kernel has also to account jointly for two additional first-order conditions. I perform empirical exercises aiming to account for equity premium in U.S. I find significant elasticity ranging from 0.24 to 0.55 and risk aversion from 1.82 to 3.51. This approach is also useful to account for the cross-section behavior of domestic assets. Such investor decisions are playing a relevant role in terms of completing markets.

KEYWORDS: Household underlying principles, Representative agent, Garnishment of a delinquent household endowment, Asset pricing puzzles, Credit market issues.

JEL CODES: C36, D14, G12.

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IN THIS PAPER, I analyze theoretical and empirical asset pricing implications of incorporating investor decisions on credit and default into the consumption capital asset pricing model (CCAPM).

This general equilibrium approach was originally developed and explored empirically by [Samuelson \(1969\)](#), [Rubinstein \(1974\)](#), [Hall \(1978\)](#), [Lucas \(1978\)](#), [Breedon \(1979\)](#) and [Grossman and Shiller \(1981\)](#). Over the last forty years, it has become a standard workhorse for consumption-based analysis of domestic and foreign financial markets stylized facts.

The most famous domestic financial markets anomaly is characterized by the high market Sharpe ratio, high level and volatility of stock returns, with low and relative constant interest rates and smooth consumption growth in U.S. More specifically, the Equity Premium Puzzle (EPP) is how [Mehra and Prescott \(1985\)](#) labelled the systematic failure of the standard power-utility CCAPM to account for the stock market risk premium in U.S., for reasonable preference parameters.

Somewhat disappointingly, this is a robust empirical phenomenon. Many studies using nonlinear generalized method of moments (GMM) or even based on log-linearized approach have found incompatible high values for the relative risk aversion and insignificant low values for the elasticity of intertemporal substitution in consumption of the representative agent.

More specifically with regards the values of these parameters, aiming to fit the values usually reported for the main macroeconomic and financial benchmarks, it has been traditional to use risk aversion numbers of 1 to 5, while [Lucas \(1990\)](#) has ruled out an elasticity below 0.5 as implausible. However, [Hall \(1988\)](#) argues that consumption growth is completely insensitive to changes in interest rates and, hence, the elasticity is unlikely to be above 0.1, and may be very close to zero. This statement predominantly influences the discussion about this elasticity until today, according to [Thimme's \(2017\)](#) survey. Thus, there is a contradiction between the dynamic macroeconomics literature and econometric studies which both use aggregate data in different ways.

This contradiction has motivated [Guvenen \(2006\)](#) to introduce limited participation in the stock market and allows for the heterogeneity in the elasticity. He finds an elasticity of the non-stockholder close to 0.10, and for the stockholder close to 1.07, while average estimated value is 0.48, consistent with the empirical evidence in macro-literature. He claims that this apparent inconsistency is largely a consequence of the representative agent perspective adopted in this literature.

To summarize, these robust findings using aggregate consumption data suggest the canonical consumption model's inability to account for the behavior of the representative agent. In addition to robust, this anomaly is very relevant. A reasonable willingness to substitute consumption intertemporally and across states, given that real interest rates are low on average and stable over time and across countries, is central to most macroeconomic models that try to capture the dynamics of output, investment, consumption, saving. In other words, research in macroeconomics has established the elasticity and the risk aversion as crucial for many questions ranging from the effects of monetary and fiscal policies to the determinants of long-run economic growth.

Moreover, previous research has cast doubt on whether a single asset pricing model was capable of correctly pricing the equity premium or to account for other domestic market puzzles, as time-varying volatility of stock returns and cross-sectional variation of expected returns, and in addition, even puzzles in international macroeconomics.

This perception led to the emergence of some empirical and theoretical strategies with respect to: i) estimation technique based on linear or nonlinear instrumental (or not) variable regressions; ii) instrument set dealing with endogeneity and the predictability of macroeconomic and financial variables; iii) the data used, i.e. sample of assets, time period and frequency or geographical origin of the data and iv) assumptions on frictions as liquidity constraints or stock market participation.

This literature suggests the studies in macro-finance as more promising to explain such stylized facts. According to [Cochrane's \(2017\)](#) survey, one of the most interesting research routes explores the consumer's utility function. Some prominent examples are catching up with the Joneses theory of [Abel \(1990\)](#), the recursive utility developed by [Epstein and Zin \(1991\)](#) and the habit formation suggested by [Campbell and Cochrane \(1999\)](#). Regarding the asset pricing implications of dealing with types of risk, one can see the effects of long-run risks proposed by [Bansal and Yaron \(2004\)](#), and idiosyncratic risks in [Constantinides and Duffie \(1996\)](#) and [Krueger and Lustig \(2010\)](#).

Some studies have analyzed only consumption data. [Yogo \(2006\)](#) addresses the relevance of disaggregating consumption among services, non-durable and durable goods, while [Ferreira and Moore \(2015\)](#) analyze the role of the variance of foreign and domestic consumptions, for instance.

More recently, [Kroencke \(2017\)](#) analyzes the properties of consumption data without measurement error and filtering and quantifies implications for asset pricing puzzles.

In this context, [Brandt et al. \(2006\)](#) claim that what we lack is a proper consumption-based model yet to be written, which, if found, could solve these asset pricing puzzles. According to [Cochrane \(2001\)](#), the central task of financial economics is to figure out what are the real risks driving asset prices and the investors' first-order conditions for savings and portfolios choice is seen as the starting point. Here I follow this quote literally. Rather than dream up models, test them and reject them, I add to this debate by having worked backwards to some extent: I propose a fresh look into the most primitive choices of the representative agent.

I allow the household to decide not only how to save, consume and invest. He also chooses how to borrow and if he will default on this obligation. As a consequence, I derive an extended system of Euler equations which incorporates two additional first-order conditions. It does not alter the fundamental asset pricing equation, does not involve higher complexity and exempts me from making additional assumptions.

I do not claim originality in addressing credit and default decisions in a general equilibrium framework. This approach has been used for normative and positive analysis in [Chatterjee et al. \(2007\)](#), [Gerlach-Kristen and Merola \(2018\)](#) and [Athreya et al. \(2018\)](#), for instance. However, this paper is the first, to my knowledge, to address the relationship between decisions on credit and default and asset pricing in a consumption-based framework.

Theoretically, this approach enables me to discuss the role of issuing an unsecured debt at t and the possibility of a delinquent behavior at $t + 1$ given a known punishment in terms of endowment garnishment. The investor can choose the payoff after the state of nature was revealed since he is allowed to decide whether he wants to repay his debt as promised or to skip debt payments without seeking formal bankrupt protection. This last option is called delinquency, a kind of informal default.

I claim that it works as an additional and versatile security able to span contingent claims. This is not an Arrow security whose payoffs are contingent on the realization of the aggregate shock, but a security with endogenous payoff. Another theoretical perspective is assuming that this framework is able to capture the rationality that investors are not particularly scared of the wealth or

consumption effects of holding stocks. They know that stocks do badly at particular times and states of nature and, hence, it is useful to introduce credit and default decisions working as a recession state variable. Not coincidentally, from 1987:1 to 2018:1, delinquency rates have increased during NBER recessions in U.S., especially in the most recent one with more than 1.1% in 18 months, and the correlation of household loan growth with excess return on stock market is -0.19.

In practice, I aim to figure out the value of assets through estimating a nonlinear extended system of Euler equations characterized by consumption-saving-portfolio-loan-delinquency decisions. I implement an empirical exercise to account for equity premium in U.S. from 1987:1 to 2018:1. I avoid the endogeneity issues discussed in [Hall \(1988\)](#). I also deal with [da Costa et al.'s \(2016\)](#) argument on the contrasting performance of the asset pricing tests in level and in its log-normalized versions.

To summarize the main empirical findings, estimating the extended system of Euler equations enables me to find individual and jointly significant parameters of two usual preferences given four different instrument sets with elasticity estimates between 0.24 and 0.55 and risk aversion estimates lower than 3.51. There seems to be no sign orthogonality misspecification. However, when the model omits household decisions on loan and default, most individual parameters are statistically zero, at the 5% significance level. As a robustness check, I propose accounting for other domestic market stylized facts that usually escape consumption-based approaches by pricing correctly returns on the six [Fama and French \(1993\)](#) dynamic portfolios formed on size and book-to-market. Once more, I am able to find reasonable and significant values for the main parameters. The evidence I report motivates the exploration of the role played by decisions on credit and default as ways to circumvent empirical difficulties of consumption-based model in a representative agent perspective.

Concerning the discussion on credit, I find that the parameter related to the garnishment of a delinquent household endowment ranges from 0.23 to 0.24. This framework enables me to draw bounds for the household decisions on loan and delinquency useful to better understanding credit cycles. I can also propose a new rule of thumb relating preferences parameters and credit variables.

This paper is structured as follows. In the Section I there is a review the literature, while Section II illustrates the setup of the model. Section III analyzes the dataset and reports main findings. Section IV discuss implications for the credit market and Section V is devoted to final remarks.

I. Literature on General Equilibrium with Credit and Default

Addressing credit decisions and its restrictions based on a neoclassical growth model with uninsurable shocks in a general equilibrium framework is not a novelty. One can see some of the early theoretical contributions in [Imrohoroglu \(1989\)](#) and [Aiyagari \(1994\)](#), for instance.

Aiming to add to the related positive discussion, [Chatterjee et al. \(2007\)](#) analyze a general equilibrium model with unsecured consumer credit that incorporates the main characteristics of U.S. consumer bankruptcy law. Their calibration exercise is useful to characterize default behavior in terms of earnings for a given set of household features. Using Dynamic Stochastic General Equilibrium (DSGE) model, [Gerlach-Kristen and Merola \(2018\)](#) find that when indebted households in Ireland start facing credit constraints, due to falling property prices, they stop smoothing consumption and start deleveraging.

In terms of normative properties, [Dávila et al. \(2012\)](#) suggest a characterization of constrained efficiency based on the first-order condition of a constrained planner's problem. It enables them to illustrate the dynamics of accumulation of capital in steady state in U.S.

Concerning more recent discussion specifically on delinquency and bankruptcy, [Athreya et al. \(2015\)](#) propose measuring the relative roles of recent bankruptcy reform and labor market risk in accounting for consumer debt and default over the Great Recession. [Mitman's \(2016\)](#) model explains the observed cross-state variation in consumer delinquency rates and it is also useful to measure the impact on bankrupt rates of two major debt-relief policies in U.S. In [Athreya et al. \(2018\)](#), the authors develop a quantitative model of debt delinquency and bankruptcy aiming to accommodate the dynamics informal consumer debt default.

To summarize, this theoretical and empirical agenda on credit and default decisions based on general equilibrium seems to be consistent and promising. However, probably due to the more contained evolution in credit before the 80's or the lack of available data on default on loans (from 1987) and on credit card (from 2001), for instance, this research agenda of decisions on credit and default has not been prioritized by asset pricing researchers.

Only recently, the severity of the last financial crisis and the following deep recession has revived interest in the links between asset prices and credit market.

In [Bordo and Jeanne \(2002\)](#), the authors suggest an interesting pass-through, according to which higher credit availability boosts asset prices through liquidity and the expectation of further rises in these prices motivates raising debt. However, during periods of falling asset prices – useful as collateral – one can expect expenditure cut back and borrowing reduction.

Another relevant contribution in this context is [Chen et al. \(2012\)](#). They propose using a multivariate analysis accounting for the phase shift mechanism, which enables them to identify causality between financial cycles and business cycles even with raw data at different frequencies. According to their main findings, at the business cycle frequency for U.S. over the period from 1965:1 to 2010:3, real output and real stock prices tend to lead effective federal funds rate and total credit in a pro-cyclical fashion.

Even more aligned to my purpose is [Berndt \(2015\)](#). This author uses reduced-form models of default calibrated to expected default losses and co-movements between default losses and an equity-based pricing kernel considering market frictions. He aims to account for historical Credit Default Swaps (CDS) spreads, while I address here the specific relationship between non-state-contingent debt, state-contingent default and consumption-based asset pricing.

For my purpose to incorporate household decisions on how to borrow and whether skipping debt payments given observable finance rate on loans, I analyze the evolution over time of these variables reported in [Figure I](#). These time series were extracted from Federal Reserve Economic Data (FRED).

During the period from 1987:1 to 2018:1 real finance rate on personal loans ranges from 1.21% to 3.93%. During NBER U.S. recessions (highlighted in the shaded areas) the volatility of these rates ranges from 0.30% to 1.03%. However, during periods without recession its standard deviation ranges from 0.19% to 0.31%. With respect to delinquency rates, its average throughout the period is 3.25%. What is striking is the growing trend in crises. These rates have increased during such recessions, especially in the most recent one – more than 1.1% in 18 months. Moreover, during this last recession delinquency rate on single-family residential mortgages have increased more than 5%, reaching a level higher than 8% right after the recession and higher than 11% in 2010.¹

¹ This variable is not used in the empirical exercise performed here and is not reported in [Figure I](#).

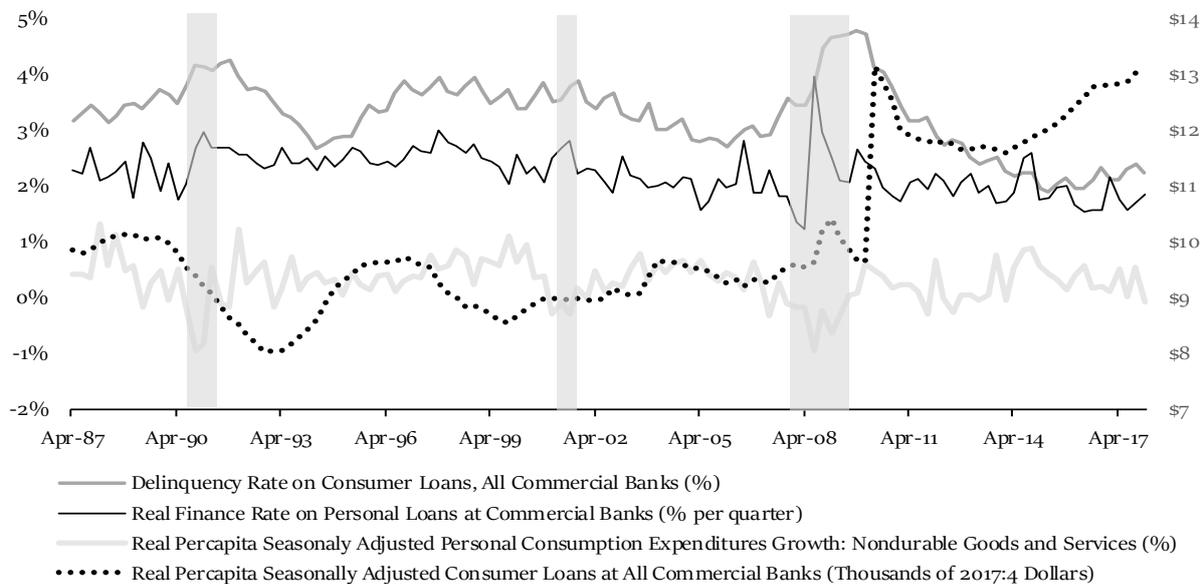


Figure I. Household loan and consumption. This figure presents the personal consumption growth and volume, finance rate and delinquency of household loans for U.S. over the period from 1987:1 to 2018:1. Data source: FRED.

Concerning personal loan growth, I may highlight the high volatility during the last recession, 2.9% and mainly after such crises, with a standard deviation of 6.3%. Meanwhile, consumption growth volatility reaches its highest value during the first recession, 0.43%.

These summary statistics, the pass-through between credit level and asset prices suggested by [Bordo and Jeanne \(2002\)](#) and the causality between financial cycles and business cycles identified in [Chen et al. \(2012\)](#) motivate me to better understand the relationship between credit decisions and asset prices. The main difference in relation to such contributions is the framework used and consequently the theoretical and empirical analysis in terms of asset pricing implications.

A. My contribution

A key contribution of this paper is to establish a connection between the U.S. aggregate data with the theory of representative household behavior that macroeconomists routinely use. Turning first to the theory, I analyze an environment where households with infinitely long planning horizons choose not only how much to consume, save and invest. I introduce the household debt and delinquency decisions into a standard model of lifecycle consumption-saving-investment.

In the original setup of this kind of general equilibrium model, the investor has to decide his current and future values of consumption and how to buy or sell of the assets available. Households face uninsured idiosyncratic and therefore have a motive to accumulate assets and to sometimes borrow in order to smooth consumption. I allow him to determine how to borrow at t , given an observable finance rate on loans at t . In this case, the investor may issue one-period unsecured debt with a face value which is non-state-contingent. As standard in this literature, household issues all unsecured debt to a single lender who must compete with a large number of domestic or foreign lenders. Lenders, moreover, are perfectly diversified across borrowers in any given current state.

This diversification is relevant since borrowers have the option to default. In practice, limited commitment for the repayment of household debt comes from two options: formal laws granting a partial or complete discharge for debts under certain circumstances and informal default and renegotiation. The literature denotes the first option as bankruptcy and the second one as delinquency. In the U.S., both channels are used routinely and the usefulness of each one as a way out of debt depends on the costs and benefits available through the other.

In this context, at date $t + 1$ investor is allowed to decide whether he want to repay his debt as promised or to skip debt payments without seeking formal bankrupt protection. Since my goal is to better isolate and understand the role of credit and partial informal default, borrower can not skip the debt as promised and then seeking formal bankrupt protection. I assume that delinquency is likely to be useful to household temporarily dealing with transitory shocks, but unlikely to last long enough to justify paying the costs of formal bankruptcy.²

Faced with such actions by borrowers, lenders retain access to the legal right to seize resources from such delinquent account holders. Most prominently, lenders may garnish wages, subject to court approval. In the case of delinquency, these costs are the implicit taxes on labor supply coming from levels of wage garnishment or increased costs of rolling over debt.

² Concerning the personal bankruptcy, I recognize the existence and the relevance of this formal procedure that removes unsecured debt obligations subject to some specific cases. However, I rely on one of the main findings reported in [Athreya et al. \(2009\)](#) to rule out this possibility. They find that unsecured credit markets pass-through increased income risk to consumption in U.S., irrespective of bankruptcy policy.

I follow this literature by also imposing a punishment to partial default. In [Athreya \(2012\)](#), for instance, a fixed proportion of the income of the delinquent is garnished, regardless of the default size. I assume a more realistic characterization: the percentage of endowment available is a linear function of the default decision. I also allow the delinquent household to roll the debt under the same conditions and costs in which he grants a new residual debt.

Lastly, following the related literature on consumption-based asset pricing, my purpose here is not characterizing the equilibrium, so I do not need to impose any no-Ponzi game condition useful to guarantee such existence. My intent is characterizing the conditions under which I am able to price correctly assets, i.e. claims on a stream of payments. I analyze such conditions by estimating the value and inferring on the individual and joint significance of the preference parameters, besides overall fitting of non-linear preference-based models.

II. Economic theory

The single most important advance in asset pricing from an economist's perspective was the development of the CCAPM associated with [Rubinstein \(1974\)](#), [Lucas \(1978\)](#), [Hall \(1978\)](#), [Breedon \(1979\)](#) and [Grossman and Shiller \(1981\)](#). Concerning the hypothesis to derive a consumption-based pricing kernel, one has not to assume complete markets or the existence of a representative consumer. First-order conditions yet to be derived apply to each individual investor and for each asset to which he has access. However, from an empirical perspective, one may assume the existence of a representative investor, given the need to use aggregate consumption data in the utility function.

If markets are complete and agents have von-Neumann Morgenstern preferences, aggregation is not needed for the existence of a representative agent. According to [Constantinides \(1982\)](#), with complete markets, the implied function emulates in the aggregate demand function all the properties implied by the choices of a rational agent. This is true even if individuals are heterogeneous in preferences and wealth, provided that they have the same time discount rate and beliefs.

However, in a more reasonable characterization of the world with incomplete markets, unless we are able to make strong assumptions about the distribution of preferences or income throughout the economy, there is little we can do to test empirically any representative agent approach.

Still with respect to the hypothesis, I follow the didactic [Cochrane's \(2001\)](#) contribution. He argues that one does not have to assume anything on returns distribution and this pricing equation must hold for any asset, stock, bond, option, etc. This fundamental relation holds for any two periods of a multiperiod model and one does not have to assume i.i.d. returns over time. I do not assume that investors have no nonmarketable human capital or no outside sources of income.

A. Canonical CCAPM

Consider an economy endowed with an infinitely lived representative consumer whose preferences are representable by a von Neumann-Morgenstern utility function $u(\cdot)$ twice continuously differentiable. The canonical problem of the representative agent is given by intertemporal and cross-state decisions on consumption, saving and investment according to

$$\text{Max}_{\xi} u(C_t) + \mathbb{E}_t[\sum_{j=1}^{+\infty} e^{-\delta j} u(C_{t+j})] \text{ s. t.} \quad (1)$$

$$C_t = e_t - \mathbf{p}_t \xi_t \quad (2)$$

$$\mathbf{C}_{t+j} = \mathbf{e}_{t+j} + \mathbf{X}_{t+j} \xi_{t+j-1} - \mathbf{p}_{t+j} \xi_{t+j} \quad (3)$$

This is a convenient mathematical formalism to model what an investor wants. Let me formalize it in the following way. $\mathbb{E}_t(\cdot)$ denotes the conditional expectation given the available information at t . The timing is such that at t , household decides current consumption, C_t , given his current endowment, e_t and current vector of N asset prices available, \mathbf{p}_t . Future consumption, \mathbf{C}_{t+j} , is random. The agent does not know his future endowment which is contingent on S possible states of nature, as well as $S \times N$ matrix of assets payoffs, given by \mathbf{X}_{t+j} .

The utility function captures the fundamental desire for more consumption, rather than posit a desire for other purposes. More specifically, the objective function (1) captures investors' impatience, by discounting the future by $e^{-\delta}$. I can quantitatively correct for delay of cash flows by estimating δ , the consumer's subjective time discount rate. Also, I need to worry about the parameters associated with each preference specification.

Now, assume that investor can freely trade as much of the assets as he wishes given the unique prices. Denote by ξ_t^i the amount of the asset i investor chooses to buy at time t . Substituting the constraints into the objective function and setting the derivative with respect to portfolio equal to zero, I obtain the first-order condition for an optimal consumption-saving-portfolio choice,

$$1 = \mathbb{E}_t \left[e^{-\delta} \frac{u'(C_{t+1})}{u'(C_t)} \mathbf{R}_{t+1}^i \right], \forall i = 1, \dots, N \quad (4)$$

where \mathbf{R}_{t+1}^i is the real gross return on asset i at $t + 1$ given by \mathbf{x}_{t+1}^i/p_t^i . This is the well-known consumption-based version of fundamental asset pricing equation. In this micro fundamented description of the world, the stochastic discount factor (SDF), \mathbf{M}_{t+1} , is given by the discounted ratio of consumption marginal utilities.

Although this consumption-based approach is very interesting from an economist's perspective, macro-finance literature is exploring different utility functions, revisiting consumption data or modelling the endowment risks as ways to deal with the empirical difficulties of canonical CCAPM. However, in the quest towards the resolution of the EPP and other puzzles in finance, the household decisions on borrowing and delinquency have not been introduced into standard general equilibrium model of lifecycle consumption-saving-investment. I address this specific issue by providing theoretical and empirical asset pricing implications of deriving an extended system of Euler equations which incorporates two additional first-order conditions. I now discuss each of the additional assumptions credit decisions in detail to highlight and explain the timing of the model.

B. Revisiting CCAPM: Model of Lifecycle Consumption-Saving-Investment-Debt-Delinquency

First, following the canonical model, it is required that investor still lives infinite periods, receives stochastic endowments and has free portfolio formation. Second, I assume that an investor wishing to borrow at date t may issue one-period unsecured debt with a non-state-contingent face value given by b_t . As standard in this literature, household issues all unsecured debt to a single lender given a known finance rate r_t . This assumption is identical to the one usually assumed in most of the studies which address credit in a general equilibrium framework.

Third, I need to introduce delinquent behavior into this quantitative model of lifecycle consumption-saving-investment-debt. At the following period, $t + 1$, the capitalized debt is $b_t \cdot (1 + r_t)$. Following [Athreya \(2012\)](#), for instance, the investor has the possibility of default, i.e. paying $b_t \cdot (1 + r_t) \cdot (1 - \mathbf{d}_{t+1})$. In other words, the household has two options: repaying debts as promised, $\mathbf{d}_{t+1} = 0$, or he can simply not repay the debt as promised, $0 < \mathbf{d}_{t+1} < 1$, without seeking formal bankrupt protection. This second option is called delinquency or partial default.

Fourth, I also follow the related literature by assuming that the financial system imposes a punishment against default behavior. The delinquent household faces endowment garnishment at $t + 1$ given by $\mathbf{e}_{t+1} \cdot \lambda \cdot \mathbf{d}_{t+1}$. More specifically, the percentage of endowment available is a linear function of the default decision given by $(1 - \lambda \cdot \mathbf{d}_{t+1})$.

Finally, after this punishment measured by the parameter λ , but still in the same date, $t + 1$, he can roll the debt $b_t \cdot (1 + r_t) \cdot \mathbf{d}_{t+1}$ under the same conditions in which he grants a new residual debt, given by \mathbf{b}'_{t+1} . In other words, at $t + 1$ household decides a total debt level, given by

$$\mathbf{b}_{t+1} = b_t \cdot (1 + r_t) \cdot \mathbf{d}_{t+1} + \mathbf{b}'_{t+1} \quad (5)$$

In this more realistic context, I must replace original budget restrictions (2) and (3) at $t + 1$ for

$$\mathbf{C}_t = e_t - \mathbf{p}_t \boldsymbol{\xi}_t + b_t \quad (6)$$

$$\mathbf{C}_{t+1} = \mathbf{e}_{t+1}(1 - \lambda \cdot \mathbf{d}_{t+1}) + \mathbf{X}_{t+1} \boldsymbol{\xi}_t - b_t \cdot (1 + r_t) \cdot (1 - \mathbf{d}_{t+1}) + \mathbf{b}'_{t+1} - \mathbf{p}_{t+1} \boldsymbol{\xi}_{t+1} \quad (7)$$

Substituting (5) in (7), after rolling the debt, one can rewrite budget constraint at $t + 1$ as

$$\mathbf{C}_{t+1} = \mathbf{e}_{t+1}(1 - \lambda \cdot \mathbf{d}_{t+1}) + \mathbf{X}_{t+1} \boldsymbol{\xi}_t - b_t \cdot (1 + r_t) + \mathbf{b}_{t+1} - \mathbf{p}_{t+1} \boldsymbol{\xi}_{t+1} \quad (8)$$

Once more, substituting the constraints into the objective function and setting the derivative with respect to b_t and \mathbf{d}_{t+1} equal to zero, I obtain the following respective first-order conditions for an optimal consumption-saving-portfolio-debt-delinquency choice,

$$1 = \mathbb{E}_t \left[e^{-\delta} \frac{u'(\mathbf{C}_{t+1})}{u'(\mathbf{C}_t)} (1 + r_t) \cdot (1 - \mathbf{d}_{t+1}) \right] \quad (9)$$

$$0 = \mathbb{E}_t [e^{-\delta} u'(\mathbf{C}_{t+1}) (b_t \cdot (1 + r_t) - \lambda \cdot \mathbf{e}_{t+j})] \quad (10)$$

Based on this extended set of household choices, I propose estimating the system of Euler equations given by fundamental asset pricing equation (4) for each one among N assets available in the economy, which remains unchanged. In addition, the consumption-based pricing kernel must also account for first-order conditions (9) and (10). First-order condition (9) applies to each individual investor and for the unique debt option to which he has access. Comparing it with condition (4), here I need an adjusted (by non-delinquency) stochastic discount factor to price real gross rate on the consumer loan.

Finally, I must highlight the contingent nature of such decisions. Issuing at time t one-period unsecured debt is non-state-contingent while at $t + 1$ he faces the possibility of a state-contingent default. In this sense, I claim that given this property it works as a versatile asset, which can be useful to span or synthesize contingent claims and thus completing markets.

This concludes my description of the physical environment.

III. Revisiting Domestic Market Stylized Facts

[Kocherlakota \(1996\)](#) assess theoretical and empirical attempts to address stock market premium in U.S. The author argues that while there are some plausible explanations for the low level of short-term Treasury Bill returns, the large equity premium is still a puzzle to finance.

In this context, there are two relevant tests that can be performed in a GMM setup. Following the related literature, first I use the canonical procedure by testing the conditional moment restrictions associated with (4) aiming to price jointly real gross return on S&P 500 and 90-day Treasury Bill in U.S. For my purposes to evaluate the extended system of first-order conditions in the second test, I just add moment restrictions (9) and (10). This pricing test procedure enables me to infer about the absolute performance of the extended system of moment conditions. It also allows us to compare the main results with those found using the canonical system. The results of both testes are reported in [Tables II](#) and [III](#), respectively.

Reassured that the extended system of Euler equations does a good job in accounting for equity premium for reasonable preference parameters and overall fitting, I proceed to a second exercise.

To account for the cross-section behavior of domestic assets, I also perform pricing tests for the six [Fama and French \(1993\)](#) benchmark dynamic portfolios extracted from the Fama–French library. These results are reported in [Table IV](#). For both exercises, I consider two consumption-based pricing kernels. Their specifications are described in the following subsection.

A. Preferences Specification

Under a consumption-based approach, it is usual to estimate the utility function parameters and then to test the associated system of orthogonality restrictions using GMM. I use as a benchmark the standard model of consumer preferences (CRRA), which is explored empirically by the majority of articles about elasticity in consumption. This preference it is scale-invariant and if agents have different levels of wealth but the same utility, then this will also be the utility of the representative agent. The SDF of this utility (M_{t+1}^{CRRA}) is given by

$$M_{t+1}^{CRRA} = e^{-\delta} \left(\frac{c_{t+1}}{c_t} \right)^{-\frac{1}{\psi}}, \text{ if } \psi \neq 1 \text{ and } M_{t+1}^{CRRA} = e^{-\delta} \frac{c_t}{c_{t+1}}, \text{ if } \psi = 1 \quad (11)$$

Although it has been extensively used in finance due to its empirical, analytical and intuitive convenience, it really does not work well in practice, there being evidence of its incapability to account for stylized facts.

Likely, the main disadvantage of (11) is that the Arrow-Pratt measure of relative risk aversion, γ , is the inverse of the agents' elasticity of intertemporal substitution in consumption parameter, ψ . Thus, allowing for high risk aversion to account for the equity premium implies to accept a very low intertemporal marginal rate of substitution. As a consequence, whenever one is willing to accept the high coefficient of relative risk aversion that is needed to correctly price the equity premium, the risk-free puzzle arises.

To avoid this trap [Epstein and Zin \(1991\)](#) define more general preferences which preserve many of the attractive features of power utility as the scale-invariance and disentangle both parameters. One of the most used specifications of the SDF of this utility (M_{t+1}^{EZIN}) is given by

$$M_{t+1}^{EZIN} = e^{-\delta\theta} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} \mathbf{R}_{w,t+1}^{\theta-1} \quad (12)$$

where $\theta = \frac{1-\gamma}{1-\psi^{-1}}$ and $\mathbf{R}_{w,t+1}$ denotes the return on the wealth portfolio.

B. Data and summary statistics

In terms of sample size, the main limitation for the time-series span used here regards the credit variables. Delinquency rate on consumer loans are available only since 1987, for instance.

The largest sample covers the period from 1978:1 to 2018:1, at a quarterly frequency, comprising 125 observations. Macroeconomic and financial variables were extracted from Federal Reserve Economic Data (FRED). Except for the delinquency rate all nominal variables were transformed into real using the corresponding U.S. Consumer Price Index.

[Table I](#) contains some summary statistics for all exogenous and endogenous variables.

The average risk premium on S&P 500 is 5.2% at an annual rate, a little lower than 6% observed in annual U.S. data from 1889–1978 originally studied by [Mehra and Prescott \(1985\)](#) and [Kocherlakota \(1996\)](#).

As usual, the correlation of per capita consumption growth with stock returns (0.11) is only slightly bigger than its correlation with bond returns (-0.05). However, the correlation of per capita consumption growth with the real gross rate on personal loan discounted by delinquency is higher, 0.35.

According to previous statistics reported in this literature, all six Fama/French dynamic portfolios formed on size and book-to-market show higher values of mean and volatility than S&P500.

I emphasize the order of magnitude of the volatility of loan and delinquency endogenous variables, a crucial element in the extended household decisions framework, characterized by first-order conditions (9) and (10).

Table I
Summary statistics

Quarterly series from 1987:1 to 2018:1 (125 observations). Data source: FRED.

	Mean	St. dev.
Exogenous variables - Assets		
Real Return on 3-Month Treasury Constant Maturity (% per quarter)	0.03%	0.52%
Real Return on S&P 500 (% per quarter)	1.29%	6.72%
Real Return on Fama/French portfolio #1 (% per quarter)	2.53%	12.39%
Real Return on Fama/French portfolio #2 (% per quarter)	3.57%	9.72%
Real Return on Fama/French portfolio #3 (% per quarter)	3.80%	10.65%
Real Return on Fama/French portfolio #4 (% per quarter)	3.08%	8.35%
Real Return on Fama/French portfolio #5 (% per quarter)	2.82%	7.82%
Real Return on Fama/French portfolio #6 (% per quarter)	3.03%	9.37%
Other exogenous variables		
Real Return on Stock market capitalization Wilshire - wealth proxy (% per quarter)	1.79%	7.89%
Real Finance Rate on Personal Loans at Commercial Banks (% per quarter)	2.22%	0.40%
Real Percapita Seasonally Adjusted Personal Income (Thousands of 2017:4 Dollars)	\$ 43.33	\$ 4.96
Investment to Capital Stock Ratio (%)	6.11%	0.58%
Dividend to Price Ratio (%)	2.19%	0.67%
Endogenous variables		
Delinquency Rate on Consumer Loans, All Commercial Banks (%)	3.25%	0.65%
Real Percapita Seasonally Adjusted Consumer Loans at All Commercial Banks (Thousands of 2017:4 Dollars)	\$10.02	\$1.38
Real Percapita Seasonally Adjusted Personal Consumption Expenditures: Nondurable Goods and Services (Thousands of 2017:4 Dollars)	\$31.87	\$3.23

C. GMM Estimation Setup and Instruments

In choosing how to weight the moments used for estimation, I face a trade-off between attaining full efficiency and correctly specifying the structure of the variance–covariance matrix. To implement a test to account for the EPP based on the canonical system of restrictions given by (4) and on the extended system given by (4), (9) and (10), I will use optimal GMM. I have a reasonable number of

moment conditions vis-à-vis my sample size. My estimates are produced by a procedure which performs a single weight iteration after the initial 2SLS estimates and then iterates coefficients to convergence. I use HAC (Newey-West) as the weighting matrix which is a heteroskedasticity and autocorrelation consistent estimator of the long-run covariance matrix.

As instruments, I use specific financial variables carefully chosen according to their forecasting potential. For the EPP, I follow [Hansen and Singleton \(1982\)](#) by using consumption growth and other endogenous and/or exogenous variables associated with the respective preference lagged one period. Considering the fact that expected returns and business cycles are correlated, I use stronger instruments as a robustness check, i.e., macroeconomic variables with documented forecasting ability: Dividend-Price ratio (DP) and Investment to Capital Stock ratio (IC), following [Cochrane's \(1996\)](#) procedure. Aiming to price Fama-French portfolios, I use endogenous and/or exogenous variables associated with the respective preference lagged one period.

D. Equity Premium Puzzle: Pricing Test Results

For my purposes, there are two relevant tests that can be performed in a GMM setup.

First, I analyze whether the parameters are statistically zero based on a robust Wald test on the individual and joint significance of them in these systems. The second is a standard overidentifying restrictions procedure. J test has the usual interpretation of a test of orthogonality between the errors in each moment restriction and the instruments used in GMM estimation. In other words, it is a specification test for the validity of instruments.

[Table II](#) displays the results of traditional consumption-based kernels in pricing returns related to EPP. This model omits household decisions on loan and delinquency. I find that individual parameters δ and ψ are statistically zero, at the 5% significance level, for both preferences specifications based on all of instrument sets used. Only at the 10% significance level, I am able to find some non-zero values for ψ . The estimation of θ in [Epstein and Zin \(1991\)](#) preferences are significant at 1% significance, except for the first instrument set. Also, Wald-test results show almost no sign of joint significance for CRRA model, while all parameters seem to be jointly statistically non-zero at 1% level in Epstein and Zin preferences.

Table II
Equity Premium Puzzle: Estimating Euler system of equations Model of Lifecycle
Consumption-Saving-Investment

Hansen's (1982) efficient GMM (update weights once and then iterate coefficients to convergence) used to test Euler equations and to estimate parameters over the period from 1987:1 to 2018:1. The moment restrictions are:

$$1 = \mathbb{E}_t \left[e^{-\delta} \frac{u'(C_{t+1})}{u'(C_t)} R_{t+1}^{SP} \right]$$

$$0 = \mathbb{E}_t \left[e^{-\delta} \frac{u'(C_{t+1})}{u'(C_t)} (R_{t+1}^{SP} - R_{t+1}^{Tbill}) \right]$$

Instrument sets 1 to 3 are comprised by variables associated with the model in question: 1) endogenous lagged one period; 2) exogenous variables lagged one period; 3) endogenous and exogenous variables lagged one period. Instrument set 4 is composed by *IC* and *DP* lagged on period. Respective p-values are reported in the box brackets. Data source: FRED. * p-value < 0.05. ** p-value < 0.01.

Parameters	Preference - CRRA				Preference - Epstein and Zin			
	Instrument sets				Instrument sets			
	#1	#2	#3	#4	#1	#2	#3	#4
δ	0.0004 [0.8988]	-0.0089 [0.2989]	0.0023 [0.3437]	-0.0061 [0.0922]	0.0187 [0.2454]	-0.0031 [0.5604]	0.0019 [0.5076]	-0.0104 [0.7114]
ψ	1.1301 [0.3559]	0.2147 [0.0719]	0.7759 [0.1115]	0.4715 [0.0720]	-0.1786 [0.2419]	0.3707 [0.0815]	0.7106 [0.0931]	0.4332 [0.7444]
θ					-2.1997 [0.2624]	0.9744 ** [0.0000]	0.9669 ** [0.0000]	0.4495 ** [0.2670]
Overall fit	0.0271	0.0415	0.0582	0.0526	0.0052	0.0405	0.0530	0.0492
(J-statistic)	[0.9865]	[0.9998]	[0.9999]	[0.9997]	[0.9423]	[0.9999]	[1.0000]	[0.9971]
Wald test - H0: $\delta=0,$ $\psi=0, \theta=0$	1.2627 [0.5319]	54.0157 ** [0.0000]	2.6350 [0.2678]	83.7831 ** [0.0000]	103.6811 ** [0.0000]	661.6879 ** [0.0000]	626.2023 ** [0.0000]	29.7864 ** [0.0000]

In Table III, I present the estimates of the parameters of the model incorporating household primitive decisions on loan and delinquency. To summarize, all parameters in both preferences frameworks using any of the instrument sets are individual and jointly significant at 1% or 5% level. Regarding *J* test, I cannot reject the null of the suitability of the model. Concerning δ slightly negative, although because people use to prefer earlier utility, it is straightforward that present values can converge with negative discount rates, as argued in Kocherlakota (1990).

Table III

Revisiting Equity Premium Puzzle: Estimating Euler system of equations Model of Lifecycle Consumption-Saving-Investment-Debt-Delinquency

Hansen's (1982) efficient GMM (update weights once and then iterate coefficients to convergence) used to test Euler equations and to estimate parameters over the period from 1987:1 to 2018:1. The moment restrictions are:

$$1 = \mathbb{E}_t \left[e^{-\delta} \frac{u'(C_{t+1})}{u'(C_t)} R_{t+1}^{SP} \right]$$

$$0 = \mathbb{E}_t \left[e^{-\delta} \frac{u'(C_{t+1})}{u'(C_t)} (R_{t+1}^{SP} - R_{t+1}^{Tbill}) \right]$$

$$1 = \mathbb{E}_t \left[e^{-\delta} \frac{u'(C_{t+1})}{u'(C_t)} (1 + r_t) \cdot (1 - d_{t+1}) \right]$$

$$0 = \mathbb{E}_t [e^{-\delta} u'(C_{t+1}) (b_t \cdot (1 + r_t) - \lambda \cdot e_{t+j})]$$

Instrument sets 1 to 3 are comprised by variables associated with the model in question: 1) endogenous lagged one period; 2) exogenous variables lagged one period; 3) endogenous and exogenous variables lagged one period. Instrument set 4 is composed by *IC* and *DP* lagged on period. Respective p-values are reported in the box brackets. Data source: FRED. * p-value <0.05. ** p-value <0.01.

Parameters	Preference - CRRA				Preference - Epstein and Zin			
	Instrument sets				Instrument sets			
	#1	#2	#3	#4	#1	#2	#3	#4
δ	-0.0051 ** [0.0000]	-0.0058 ** [0.0000]	-0.0043 ** [0.0000]	-0.0057 ** [0.0000]	-0.0162 ** [0.0000]	-0.0059 ** [0.0000]	-0.0066 ** [0.0000]	-0.0203 * [0.0396]
ψ	0.5497 ** [0.0000]	0.4368 ** [0.0000]	0.5238 ** [0.0000]	0.4993 ** [0.0000]	0.4298 ** [0.0000]	0.5148 ** [0.0000]	0.4769 ** [0.0000]	0.2383 ** [0.0004]
λ	0.2354 ** [0.0000]	0.2365 ** [0.0000]	0.2371 ** [0.0000]	0.2380 ** [0.0000]	0.2363 ** [0.0000]	0.2354 ** [0.0000]	0.2370 ** [0.0000]	0.2401 ** [0.0000]
θ					0.8249 ** [0.0000]	0.9352 ** [0.0000]	0.8983 ** [0.0000]	0.7854 ** [0.0004]
Overall fit	0.1843	0.1862	0.2001	0.1520	0.1805	0.1903	0.1980	0.1530
(J-statistic)	[1.0000]	[1.0000]	[1.0000]	[1.0000]	[1.0000]	[1.0000]	[1.0000]	[1.0000]
Wald test - H0: $\delta=0$, $\psi=0$, $\lambda=0$, $\theta=0$	7.7×10^3 ** [0.0000]	2.8×10^4 ** [0.0000]	2.6×10^5 ** [0.0000]	1.4×10^4 ** [0.0000]	1.2×10^4 ** [0.0000]	2.8×10^4 ** [0.0000]	3.0×10^4 ** [0.0000]	7.8×10^3 ** [0.0000]

How relevant are these findings and what are they saying to us? In few words, the values of the significant parameters are closer to the values expected by theory and by calibration exercises.

Using simulated data based on sample path of more than 60,000 observations, [Guvenen \(2006\)](#) replicates existing studies as [Hansen and Singleton \(1983\)](#) by proposing to estimate the elasticity from the log-linearized consumption Euler using instruments commonly used in the literature. When he includes the time-varying conditional variance, the aggregate elasticity estimate is 0.48, while the elasticity estimates are 0.10 and 1.07 for non-stockholder and stockholder, respectively.

However, when the Euler equation is estimated without any sources of bias, the resulting estimate of the elasticity, which is an average of individual elasticities, is unlikely to be significant and above 0.1 for the representative agent, as reported in the recent [Thimme's \(2017\)](#) survey.

Comparing with previous evidence, [Hansen and Singleton \(1982\)](#) find ψ close to unity, which is due to an upward bias created by endogeneity, according to [Hall \(1988\)](#). [Ogaki and Reinhart \(1998\)](#) and [Basu and Kimball \(2000\)](#) have identified possible sources able to create downward bias in the estimates of the elasticity. Both studies obtain elasticity for the representative agent around 0.35.

To summarize, this literature remains nonconsensual and contradictory, since it is unlikely to find significant elasticity close to 0.5 using the representative agent.

In the empirical exercise proposed here to account for EPP, I deal with Hall's criticize by circumventing endogeneity problem, i.e., I rely on strong exogenous instruments. However, while ψ estimates in Hall's study are insignificant avoiding endogeneity, I strongly reject individual and jointly insignificance of ψ when household also decide on loan and delinquency, given all instrument sets used here. Moreover, I follow [Lucas' \(1990\)](#) rule of thumb which relates interest rates r_f with a consumer's subjective time discount rate and consumption growth via

$$r_{f,t} = \delta + \psi^{-1} \cdot \ln(C_{t+1}/C_t) \quad (13)$$

Considering the average values of consumption growth and risk-free rate time series during the period from 1987:1 to 2018:1 and given the range of my estimates of δ using CRRA preference, the elasticity of intertemporal substitution for the representative agent should range from 0.46 to 0.61. My elasticity estimates using this traditional preference range from 0.43 to 0.55.

Considering Epstein and Zin preferences, I find significant and reasonable estimates of elasticity ranging from 0.43 to 0.51, except for the fourth instrument set.

Second, it is usual to find high and incompatible values for the coefficient of relative risk aversion even in the literature that has focused on developing models of investor behavior. I find significant values for Arrow-Pratt measure of relative risk aversion ranging from 1.82 to 2.29 when the household with CRRA preferences also decide on borrow and default. Comparing these findings with some recently reported also using CRRA, [Matos and da Costa \(2016\)](#) find this risk aversion of around 5.8 covering the period from 1977:1 to 2004:4. In [Kroencke \(2017\)](#) this parameter assumes values higher than 15.1 for a longer period from 1928 to 2014 using filtered and nonfiltered consumption data extracted from National and Income Product Accounts (NIPA). Considering Epstein and Zin preferences, I find significant and reasonable estimates of risk aversion ranging from 1.88 to 3.51.

Third, a classical issue in this literature is forecasting risk-free rate given by the inverse of unconditional expectation of the SDF. Comparing the results obtained from the standard and the extended systems of restrictions given both preferences specifications, I find the lower level of mean square error when I use the extended system of Euler equations and the consumer has a CRRA preference. This error is 0.047%, while the errors are 0.365% and 0.443% when I use canonical system with CRRA and Epstein-Zin preferences, respectively.

Finally, I find that the parameter λ , related to the garnishment of a delinquent household endowment, ranges from 0.2354 to 0.2401. Among the few similar empirical contributions, in [Athreya \(2012\)](#) this parameter is assumed to be lower than 0.1 for credit card in U.S.

E. Additional Pricing Test Results: Dynamic Portfolios

Consumption-based models predict that assets with high average returns have high consumption covariances while assets with low average returns have low covariances. To deal with this evidence, the literature following mainly [Fama and French \(1993\)](#) has sorted firms according to their market equity and book-to-market ratio. These size and value portfolios are very much the standard test assets in this asset pricing literature. Thus, another way to corroborate the success of extended version of the Euler system is looking at the results in the empirical cross-sectional pricing tests.

Table IV reports the main results of pricing test of six Fama and French (1993) portfolios.

Table IV

Revisiting other market stylized facts: Estimating Euler equations of the model of Lifecycle Consumption-Saving-Investment-Debt-Delinquency

Hansen's (1982) efficient GMM (update weights once and then iterate coefficients to convergence) used to test Euler equations and to estimate parameters over the period from 1987:1 to 2018:1. The moment restrictions are:

$$1 = \mathbb{E}_t \left[e^{-\delta} \frac{u'(C_{t+1})}{u'(C_t)} R_{t+1}^{FFi} \right], i = 1, \dots, 6$$

$$1 = \mathbb{E}_t \left[e^{-\delta} \frac{u'(C_{t+1})}{u'(C_t)} (1 + r_t) \cdot (1 - d_{t+1}) \right]$$

$$0 = \mathbb{E}_t [e^{-\delta} u'(C_{t+1}) (b_t \cdot (1 + r_t) - \lambda \cdot e_{t+j})]$$

Instrument sets 1 to 3 are comprised by variables associated with the model in question: 1) endogenous lagged one period; 2) exogenous variables lagged one period; 3) endogenous and exogenous variables lagged one period. Respective p-values are reported in the box brackets. Data source: FRED. * p-value < 0.05. ** p-value < 0.01.

Parameters	Preference - CRRA			Preference - Epstein and Zin		
	Instrument sets			Instrument sets		
	#1	#2	#3	#1	#2	#3
δ	0.0183 ** [0.0000]	0.0208 ** [0.0000]	0.0208 ** [0.0000]	0.4437 [0.9119]	0.0302 ** [0.0000]	0.0300 ** [0.0000]
ψ	0.7068 ** [0.0000]	0.7501 ** [0.0000]	0.6814 ** [0.0000]	0.0042 [0.9160]	0.1819 ** [0.0000]	0.1555 ** [0.0000]
λ	0.2354 ** [0.0000]	0.2361 ** [0.0000]	0.2362 ** [0.0000]	0.2356 ** [0.0000]	0.2358 ** [0.0000]	0.2359 ** [0.0000]
θ				0.0067 [0.9159]	0.2515 ** [0.0000]	0.2370 ** [0.0000]
Overall fit	0.1914	0.2034	0.2050	0.1875	0.2040	0.2054
(J-statistic)	[1.0000]	[1.0000]	[1.0000]	[1.0000]	[1.0000]	[1.0000]
Wald test - H0: $\delta=0$, $\psi=0$, $\lambda=0$, $\theta=0$	1.4×10^4 ** [0.0000]	1.5×10^5 ** [0.0000]	1.1×10^7 ** [0.0000]	2.0×10^4 ** [0.0000]	1.2×10^5 ** [0.0000]	1.6×10^6 ** [0.0000]

In this robustness exercise, I reject at 1% significance level the null that parameters are individual zero for both preferences, except for Epstein and Zin model with first instrument set comprised by only endogenous variables. I reject the null of joint insignificance for all cases. With regards to the overall fitting, I cannot reject the null of the suitability of the model.

I find that elasticity ranges from 0.15 to 0.75, while risk aversion ranges from 1.33 to 2.29. I find positive discount rates and the values of the garnishment parameter remain similar to those previously reported. My results suggest that I am also able to account for cross-section dynamics of domestic assets in U.S, at least in terms of size and value. This is a difficult test even for the SDF mimicking portfolio statistically devised and used in [da Costa, Issler and Matos \(2015\)](#).

IV. Discussion on Credit Market

The literature on asset pricing puzzles is consensual in arguing that consumption is not volatile enough to generate the observed equity premium in the standard power-utility consumption-based model without very large risk aversion coefficients. In other words, expected returns on stock market are high because they fall when investors marginal utility is also high. Over the period from 1987 to 2018, the Sharpe ratio of market is 0.38 while annual consumption growth ranges from -3.8% to 5.3% and has a standard deviation of 0.8%. Their correlation is 0.11.

In this statistical context, loan growth may be playing a relevant role. It has a volatility of 7.5% and its correlation with excess return on stock market is -0.19. I highlight that credit cycles occur more frequently than consumption cycles. Moreover, according to [Bordo and Jeanne \(2002\)](#) and [Chen et al. \(2012\)](#), there is evidence suggesting the pass-through between credit and asset prices besides causality between financial cycles and business cycles.

In this context, I introduce credit and default decisions working as a recession state variable, which also enables me to discuss on relevant issues on credit.

According to robust and significative results reported in [Tables III and IV](#), the parameter λ , related to the garnishment of a delinquent household endowment, ranges from 0.2354 to 0.2401. This finding means that, for delinquency rates ranging from 1.88% to 4.77% throughout the period, about 0.4% and 1.1% of endowment becomes unavailable for delinquent investors in U.S.

In [Figure II](#), I plot time series of the difference between the lender’s income with garnishment of endowment and the observed default as a ratio of capitalized debt, given by $\frac{\lambda \cdot d_{t+1} \cdot e_{t+1} - b_t \cdot (1+r_t) \cdot d_{t+1}}{b_t \cdot (1+r_t)}$. This difference has decreased during NBER U.S. recessions (highlighted in the shaded areas), except for the first one. I can identify the minimum value for the garnishment parameter able to provide a positive time series of this net income, 0.291, a higher value than the values obtained in the estimates.

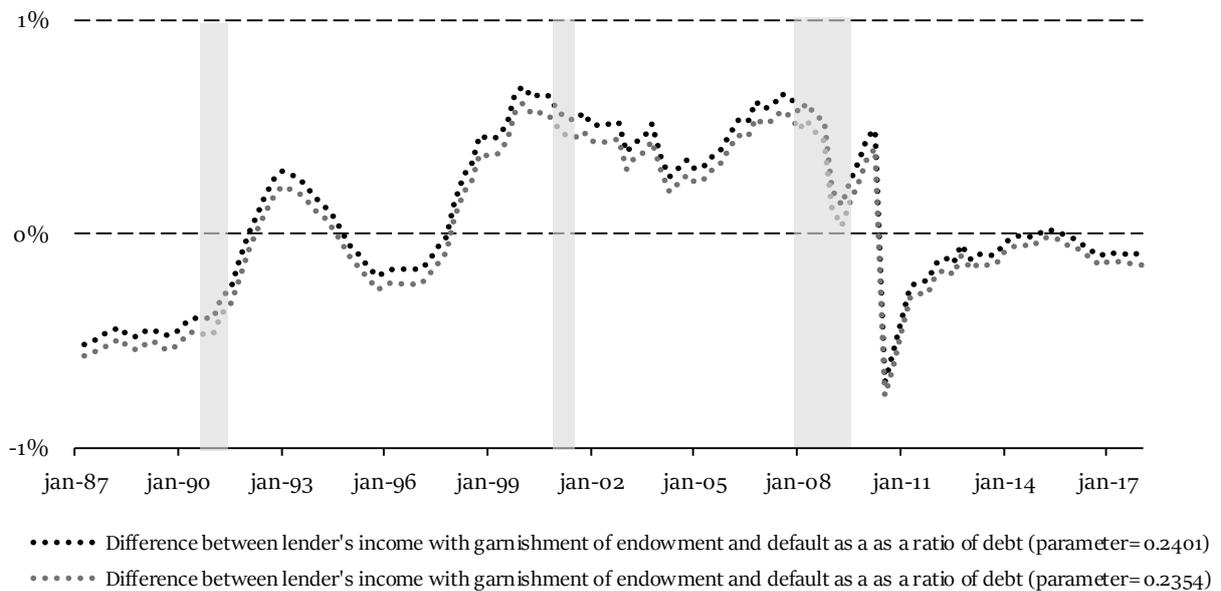


Figure II. Lender’s net income. This figure presents the difference between the lender’s income with garnishment of endowment and the observed default as a ratio of capitalized debt for U.S. over the period from 1987:1 to 2018:1.

Another interesting implication of this parameter is due to the sign of the first derivative $\partial b_t / \partial \lambda > 0$, i.e. the possibility to raise household credit grant given a higher value for the garnishment of endowment parameter, without reducing loan rates. These findings can be useful to provide support to the conduction of a policy experiment on credit regarding garnishment.

This extended version of household primary decisions also enables me to draw the locus of possible combinations of delinquency (horizontal axis) and real per capita consumer loans (vertical axis). In [Figure III](#), I plot such possible locus based on the estimates reported in [Table III](#) using Epstein and Zin preferences and the fourth instrument set comprised only by strong exogenous variables.

More specifically, I plot three set of possible ordered pairs of delinquency and loan given unconditional moments related to restrictions (9) and (10), given a range of values for finance rate from 0% to 5%. The difference among each set is only the endowment level which ranges from US\$ 35 to US\$ 52 thousands of 2017:4 dollars, minimum and maximum values respectively during the period from 1987:1 to 2018:1. Formally, the locus is given by the following equations

$$\mathbb{E}[(1 + r_t) \cdot (1 - \mathbf{d}_{t+1})] = \frac{1 - \text{cov}\left[e^{-\delta \frac{u'(C_{t+1})}{u'(C_t)}}, (1+r_t) \cdot (1-\mathbf{d}_{t+1})\right]}{\mathbb{E}\left[e^{-\delta \frac{u'(C_{t+1})}{u'(C_t)}}\right]} \quad (14)$$

$$\mathbb{E}[(b_t \cdot (1 + r_t))] = \lambda \cdot \mathbb{E}(e_{t+j}) - \frac{\text{cov}[e^{-\delta u'(C_{t+1})}, b_t \cdot (1+r_t) - \lambda \cdot e_{t+j}]}{\mathbb{E}[e^{-\delta u'(C_{t+1})}]} \quad (15)$$

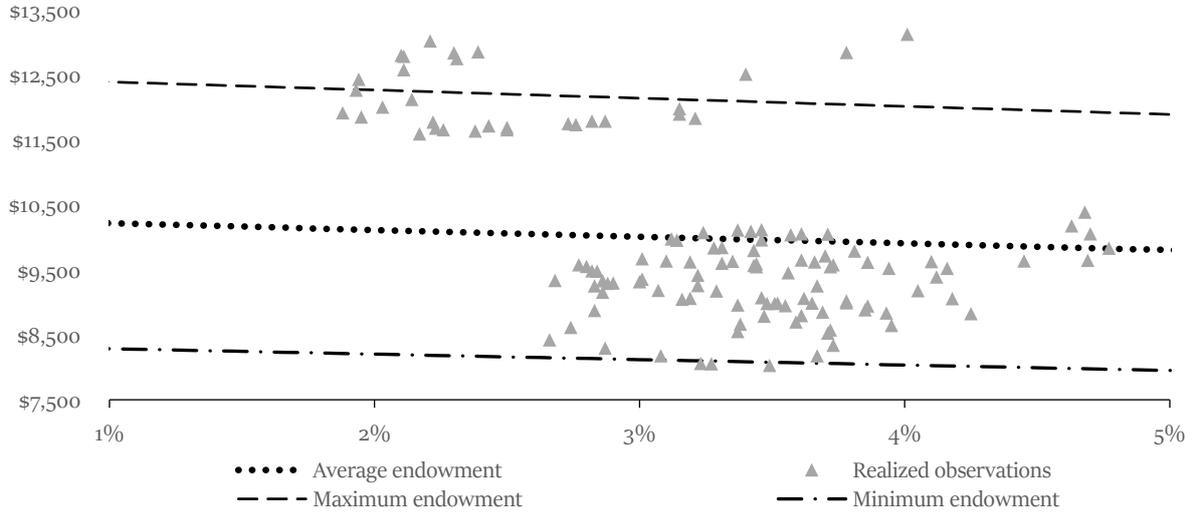


Figure III. Delinquency and Loan. This figure presents the personal loan delinquency (horizontal axis) and volume in thousands of 2017:4 dollars (vertical axis) dispersion for U.S. over the period from 1987:1 to 2018:1. It also presents credit bounds based on unconditional moments of first-order conditions (9) and (10).

This approach is useful to draw bounds for the household decisions on loan and delinquency and then better understanding the dispersion of such endogenous variables. It suggests smoother household credit cycles, mainly in scenarios with delinquency rate lower than 2.5%, and high level of loans. Moreover, this framework enables me to follow Lucas' (1990) rule of thumb, but now to relate b_t in log with a preference's parameters, endogenous and exogenous variables via

$$\ln(b_t) = -r_t + \ln(\lambda) + \ln(C_{t+1}^{-\gamma} \cdot e_{t+j}) - \ln(C_{t+1}^{-\gamma}) \quad (16)$$

Based on the average values of variables during the period from 1987:1 to 2018:1, for a range of reasonable values for risk aversion between 0 and 5, the garnishment parameter λ may range from 0.2355 and 0.2495. These limits are aligned to the values I find in all exercises performed here.

V. Concluding Remarks

In the quest towards the partial or total resolution of famous and robust financial markets stylized facts, i.e. puzzles, macro-finance literature has explored a wide range of alternative techniques, data, preferences and market structures. To summarize this extensive literature, [Cochrane \(2001\)](#) claims for the relevance of adding recession state variables as the natural route to solving such puzzles.

I enter this debate by revisiting the most primitive choices of the representative agent. I incorporate household debt and delinquency decisions, working as such recession state variables, into a standard quantitative model of lifecycle consumption-saving-investment.

This theoretical innovation does not compete with or rival the extensive literature mentioned above. On the contrary, I invite theoretical and empirical researchers who have proposed or applied such theories to revisit macro-finance puzzles by assuming this extended system of Euler equations.

Theoretically, the key is the possibility of issuing a debt and in the next period skip or not this obligation. It works as an additional, rare and versatile asset relevant in terms of market completeness due to its endogenous payoff. In practice, I am imposing that the model has to account for this asset, in addition to the assets that one wishes to price. I claim that the satisfactory empirical findings reported here come from this spam of contingent claims, rather than higher variance of the stochastic discount factor, which has been usually the purpose in many other approaches.

Any advance in the challenge of finding quantitative and theoretically explicit measures of main preference parameters that account for asset pricing facts is far from trivial. More specifically and aligned to my contribution, adding a security to span contingent claims is not necessarily relevant. [Krueger and Lustig \(2010\)](#) provide general conditions under which opening or closing down insurance markets for idiosyncratic risks has no effect on asset pricing, for instance. In short, the Equity Premium Puzzle applies to a much larger class of incomplete markets models.

In terms of future research agenda, I suggest revisiting other domestic market anomalies, and given the high volatility of credit and the levels of delinquency in emerging economies, this approach can be useful to price correctly the equity premium in these countries. Finally, [da Costa et al. \(2015\)](#) and [Matos and da Costa \(2016\)](#) argue that progresses on preference-based solutions to puzzles in domestic financial markets will also deal with puzzles in the foreign exchange market. The framework proposed here is supposed to provide researchers to combine it with any among all these research promising routes to account for international macroeconomic puzzles.

Concerning promising routes to be explored in this specific researching line, it may be useful to incorporate the role played by total informal default and bankruptcy in pricing assets in this framework proposed here. Another extension is related the punishment. In practice, delinquency costs are difficult, if not impossible, to directly observe. Given the legislation that restricts lenders from imposing nonpecuniary costs on borrowers, it is clear that these costs exist. Therefore, following this literature it is interesting to consider other possibilities, as potentially increasing costs of rolling over debt. Finally, a natural route to add to this discussion is disaggregating credit among relevant credit lines, as student loans, credit card and auto loans.

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I incorporate household debt and delinquency decisions into a standard model of lifecycle consumption-saving-investment. Deriving this extended system of Euler equations does not alter consumption-based fundamental asset pricing equation. It imposes that the pricing kernel has also to account jointly for two additional first-order conditions. I perform empirical exercises aiming to account for equity premium in U.S. I find significant elasticity ranging from 0.24 to 0.55 and risk aversion from 1.82 to 3.51. This approach is also useful to account for the cross-section behavior of domestic assets. Such investor decisions are playing a relevant role in terms of completing markets.