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The role of household debt and delinquency decisions in consumption-based asset pricing

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HIGHLIGHTS

- We revisit investor decisions by allowing him to borrow and to opt for default through delinquency
- We derive an extended system of Euler equations useful to account for domestic market stylized facts
- Our contribution is supposed to provide researchers to combine it with other approaches

ABSTRACT

We add to asset pricing theory by introducing household's debt and delinquency decisions into an otherwise standard quantitative model of lifecycle consumption-saving-investment. This extended system of Euler equations incorporates two new first-order conditions. It does not involve higher complexity, does not alter consumption-based fundamental asset pricing equation and exempts us from making additional premises. We perform two empirical exercises, one to account for equity premium in U.S. and other to price six Fama-French dynamic portfolios. We are able to find significant elasticity ranging from 0.15 to 0.75. These additional decisions may be playing a role in terms of completing markets.

JEL CODES: C36, D14, G12.

KEYWORDS:
Household underlying principles
Complete Markets
Asset pricing puzzles

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1. Introduction

IN THIS PAPER, we analyze theoretical and empirical asset pricing implications of the consumption capital asset pricing model (CCAPM) with investors' unsecured credit and default decisions.

This kind of general equilibrium approach was originally developed and explored empirically by Lucas (1978), Hall (1978), Breeden (1979) and Grossman and Shiller (1981) and is one the most important advances in finance.

This elegant framework widely used in macroeconomic theory was the natural candidate to price an asset in an equilibrium setting with risk averse agents. Over the last thirty years, it has become a standard workhorse for consumption-based analysis of financial markets stylized facts.

The most famous domestic financial markets anomaly is characterized by two empirical phenomena: the large equity premium and the low risk-free rate in U.S. More specifically, the equity premium puzzle (EPP) is how Mehra and Prescott (1985) labelled the systematic failure of the CCAPM in its canonical form to account for the stock market risk premium in U.S., for reasonable preference parameters. Somewhat disappointingly, this a robust puzzle. Many studies using mostly nonlinear generalized method of moments (GMM) have found insignificant low values for the representative agents' elasticity of intertemporal substitution in consumption. Hall (1988) claims that this parameter is unlikely to be above 0.1, and may be zero. This statement predominantly influences the discussion about this elasticity until today, according to Thimme's (2017) survey.

These robust findings suggest the traditional canonical consumption model's inability to account for its overidentifying restrictions since the elasticity is a central parameter in models of dynamic choice in macroeconomics and finance. Moreover, previous research has cast doubt on whether a single asset pricing model was capable of correctly pricing the equity premium or to account for other domestic market or international puzzles. This perception led to the emergence of some promising (or not) empirical and theoretical strategies.

Thimme (2017) summarizes studies that provide estimates of this relevant parameter. He shows that this literature emerges with respect to: i) estimation technique based on linear or nonlinear instrumental (or not) variable regressions; ii) instrument set dealing with endogeneity and the predictability of macroeconomic and financial variables; iii) the data used, i.e. sample of assets, time period and frequency or geographical origin of the data and iv) assumptions on frictions as liquidity constraints or stock market participation. Some of the most interesting research routes explore the consumer's utility function as Abel (1990), Epstein and Zin (1991) and Campbell and Cochrane (1999). Regarding specifically consumption data, Yogo (2006) addresses the relevance of disaggregating consumption among services, non-durable and durable goods, while Ferreira and Moore (2015) analyze the role of the variance of foreign and domestic consumptions, for instance.

In short, Brandt et al. (2006) claim that what we lack is a proper consumption-based model yet to be written, which, if found, could solve these asset pricing puzzles. We add to this debate by proposing a fresh look into the most primitive choices of the representative agent. We allow the household to decide not only how to save, consume and invest. He also chooses how to borrow and if he will default on this obligation. Estimating this extended system of Euler equations which incorporates two new first-order conditions does not alter fundamental asset pricing equation, does not involve higher complexity and exempts us from making additional assumptions.

We do not claim originality in addressing credit and default decisions in a general equilibrium framework. This approach has been used for normative and positive analysis in Chatterjee et al. (2007), Gerlach-Kristen and Merola (2018) and Athreya et al. (2018), for instance. However, this paper is the first, to our knowledge, to address the relationship between credit and default decisions and asset price in a consumption-based framework.

Theoretically, this approach enables us to discuss about role of issuing an unsecured debt and the possibility of a delinquent behavior given a punishment in terms of endowment garnishment. We claim that it works as a kind of additional security able to span contingent claims and then complete markets.

In practice, our purpose is just to figure out the value of assets through estimating a nonlinear system of Euler equations characterized by consumption-saving-portfolio-loan-delinquency decisions. We implement an empirical exercise to account for equity premium in U.S. from 1987:1 to 2018:1. We avoid endogeneity critique discussed in Hall (1988). We also deal with da Costa et al.'s (2016) argument on the contrasting performance of the asset pricing tests in level and in its log-normalized versions.

There seems to be no sign orthogonality misspecification and we find individual and jointly significant parameters estimates of usual preferences using different instrument sets. We find elasticity above 0.5. As a robustness check, we account for other domestic market stylized facts that usually escape consumption-based approaches. We price correctly returns on the six Fama and French (1993) dynamic portfolios formed on size and book-to-market. Once more, we are able to find reasonable and significant values for the main parameters.

We believe we have offered evidence on the theoretical and empirical role played by credit and default decisions in consumption-based asset pricing discussion.

This paper is structured as follows. In the next section we review the literature, while Section 3 illustrates the setup of the model. Section 4 analyses the dataset and reports main findings. Section 5 is devoted to final remarks.

2. Literature Review

2.1. General Equilibrium with Credit and Default

Addressing credit decisions and its restrictions based on a neoclassical growth model with uninsurable shocks in a general equilibrium framework is not a novelty.[1] Aiming to add to the related positive discussion, Chatterjee et al. (2007) analyze a general equilibrium model with unsecured consumer credit that incorporates the main characteristics of U.S. consumer bankruptcy law. Their calibration exercise is useful to characterize default behavior in terms of earnings for a given set of household characteristics. Using DSGE, Gerlach-Kristen and Merola (2018) find that when indebted households start facing credit constraints, due to falling property prices, they stop smoothing consumption and start deleveraging.

In terms of normative properties, Dávila et al. (2012) suggest a characterization of constrained efficiency based on the first-order condition of a constrained planner's problem. It enables them to illustrate the dynamics of accumulation of capital in steady state in U.S.

Concerning more recent discussion specifically on default and bankruptcy, Athreya et al. (2015) propose measuring the relative roles of recent bankruptcy reform and labor market risk in accounting for consumer debt and default over the Great Recession. Mitman's (2016) model explains the observed cross-state variation in consumer delinquency rates and it is also useful to measure the impact on bankrupt rates of two major debt-relief policies in U.S. In Athreya et al. (2018), the authors develop a quantitative model of debt delinquency and bankruptcy aiming to accommodate the dynamics informal consumer debt default.

To summarize, this theoretical and empirical agenda on credit and default decisions based on general equilibrium seems to be consistent and promising. However, probably due to the more contained evolution in credit before the 80's or the lack of available data on default on loans (from 1987) and on credit card (from 2001), this research agenda on credit and delinquency decisions has not been prioritized by asset pricing researchers.

Only recently, the severity of the last financial crisis and the following deep recession has revived interest in the links between asset prices and credit market. In Bordo and Jeanne (2002), the authors suggest an interesting pass-through, according to which higher credit availability boosts asset prices through liquidity and the expectation of further rises in these prices motivates raising debt. However, during periods of falling asset prices – useful as collateral – one can expect expenditure cut back and borrowing reduction.

Another relevant contribution in this context is Chen et al. (2012). They propose using a multivariate analysis accounting for the phase shift mechanism, which enables them to identify causality between financial cycles and business cycles even with raw data at different frequencies. According to their main findings, at the business cycle frequency for U.S. over the period from 1965:1 to 2010:3, real output and real stock prices tend to lead effective federal funds rate and total credit in a pro-cyclical fashion.

Even more aligned to our purpose is Berndt (2015). This author uses reduced-form models of default calibrated to expected default losses and co-movements between default losses and an equity-based pricing kernel considering market frictions. He aims to account for historical CDS spreads, while we address the relationship between non-state-contingent debt, state-contingent default and asset prices.

[1] See some of the early theoretical contributions in Imrohoroglu (1989) and Aiyagari (1994), for instance.

2.2. Our Contribution

For our purpose to incorporate household decisions on how to borrow and whether skipping debt payments given observable finance rate on loans, we may analyze the evolution over time of these variables reported in Figure 1.

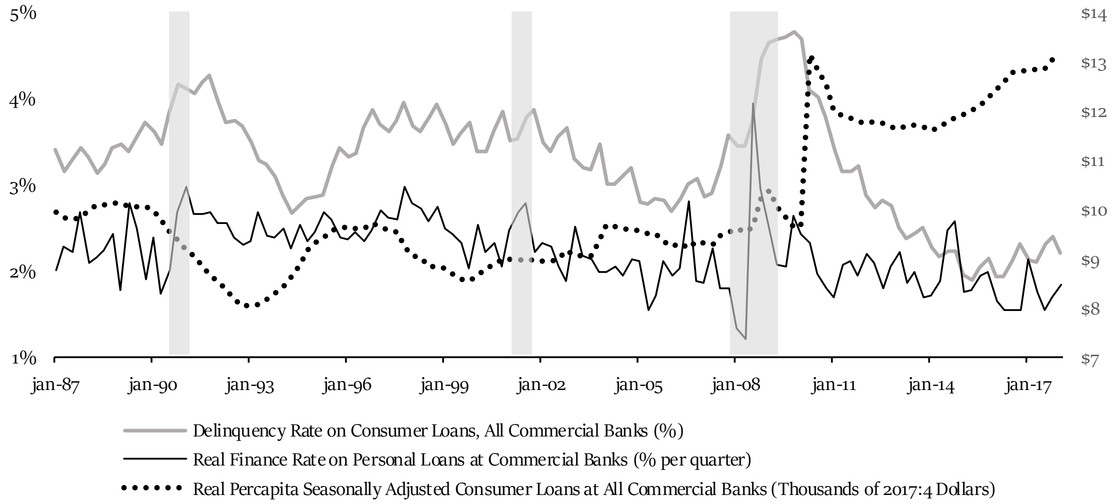

Fig. 1. Household loan: volume, finance rate and delinquency (1987:1 to 2018:1). Data source: Federal Reserve.

We can observe that during U.S. NBER recessions (highlighted in the shaded areas) delinquency rates have increased, especially in the most recent recession – more than 1% in 18 months. Finance rates seem to oscillate more intensely and apparently, consumer loan does not suggest a single visible pattern. Average delinquency rate over the period from 1987:1 to 2018:1 is 3.25% with real finance rates ranging from 1.21% to 3.93% (per quarter). The standard deviations of these series should not be negligible.

These summary statistics, the pass-through between credit level and asset prices suggested by Bordo and Jeanne (2002) and the causality between financial cycles and business cycles identified in Chen et al. (2012) motivate us to better understand the relationship between credit and asset prices. Our difference in relation to these contributions is the framework used and consequently the analysis we can do.

We introduce the household's debt and delinquency decisions into a canonical standard quantitative model of lifecycle consumption-saving-investment. In the original setup of this kind of general equilibrium model, the investor has to decide his current and future values of consumption and how to buy or sell of the assets available.

We allow him to endogenously determine how to borrow in t, given an observable finance rate on loans in t. More specifically, an investor may issue one-period unsecured debt with a face value which is non-state-contingent. In $t + 1$ investor is allowed to decide whether he want to repay his debt as promised or to skip debt payments without seeking formal bankrupt protection. This behavior is defined in this literature as delinquent.

We follow Athreya (2012) by imposing a punishment to default. The delinquent investor faces endowment garnishment: the percentage of endowment available is a linear function of the default decision. We also allow him to roll the debt under the same conditions in which he grants a new residual debt. Concerning the personal bankruptcy, we recognize the existence and the relevance of this formal procedure that removes unsecured debt obligations subject to some specific cases. However, we rely on one of the main findings reported in Athreya et al. (2009) to rule out this possibility. They find that unsecured credit markets pass-through increased income risk to consumption in U.S., irrespective of bankruptcy policy and the information possessed by lenders.

Following the related literature on consumption-based asset pricing, our purpose here is not characterizing the equilibrium, so we do not need to impose any no-Ponzi game condition useful to guarantee such existence. Our intent is characterizing the conditions under which we are able to price correctly assets, i.e. claims on a stream of payments. We analyze such conditions by estimating the value and inferring on the individual and joint significance of parameters, besides overall fitting of non-linear preference-based models.

3. Economic Theory

3.1. Canonical CCAPM

The single most important advance in asset pricing from an economist's perspective was the development of the CCAPM associated with Lucas (1978), Hall (1978), Breeden (1979) and Grossman and Shiller (1981).

Consider an economy endowed with an infinitely lived representative consumer whose preferences are representable by a von Neumann-Morgenstern utility function $u(.)$ twice continuously differentiable. The canonical problem of the representative agent is given by intertemporal and cross-state decisions on consumption, saving and investment according to

$$Max_{\xi}\, u(C_t) + \mathbb{E}_t[\sum_{j=1}^{+\infty} e^{-\delta j} u(\boldsymbol{C}_{t+j})] \text{ s. t.} \tag{1}$$

$$C_t = e_t - \boldsymbol{p}_t \boldsymbol{\xi}_t \tag{2}$$

$$\boldsymbol{C}_{t+j} = \boldsymbol{e}_{t+j} + \boldsymbol{X}_{t+j}\boldsymbol{\xi}_{t+j-1} - \boldsymbol{p}_{t+j}\boldsymbol{\xi}_{t+j}, j = 1, \dots, \infty \tag{3}$$

This is a convenient mathematical formalism to model what an investor wants. Let us formalize it in the following way. $\mathbb{E}_t(.)$ denotes the conditional expectation given the available information in t. The timing is such that in t, household decides current consumption, C_t, given his current endowment, e_t and current vector of N asset prices available, $\boldsymbol{p}_t$. Future consumption, $\boldsymbol{C}_{t+j}$, is random. The agent does not know his future endowment which is contingent on S possible states of nature, as well as SxN matrix of assets payoffs, given by $\boldsymbol{X}_{t+j}$.

The utility function captures the fundamental desire for more consumption, rather than posit a desire for other purposes. More specifically, the objective function (1) captures investors' impatience, by discounting the future by $e^{-\delta}$. We can quantitatively correct for delay of cash flows by estimating δ, the consumer's subjective time discount rate. Also, we need to worry about the parameters associated with each preference specification.

Now, assume that investor can freely trade as much of the assets as he wishes given the unique prices. Denote by ξ_t^i the amount of the asset i investor chooses to buy in t. Substituting the constraints into the objective function and setting the derivative with respect to portfolio equal to zero, we obtain the first-order condition for an optimal consumption-saving-portfolio choice,

$$1 = \mathbb{E}_t\left[e^{-\delta}\frac{u'(C_{t+1})}{u'(C_t)}\boldsymbol{R}_{t+1}^i\right], \forall i = 1, \dots, N \tag{4}$$

where $\boldsymbol{R}_{t+1}^i$ is the real gross return on asset i in $t+1$ given by $\boldsymbol{x}_{t+1}^i/p_t^i$. This is the well-known consumption-based version of fundamental asset pricing equation. In this micro fundamented description of the world, the stochastic discount factor (SDF), $\boldsymbol{M}_{t+1}$, is given by the discounted ratio of consumption marginal utilities.

Concerning the hypothesis to derive a consumption-based pricing kernel, we have not assumed complete markets or the existence of a representative consumer. First-order condition (4) applies to each individual investor and for each asset to which he has access. However, from an empirical perspective, one may assume the existence of a representative investor, given the need to use aggregate consumption data in $u(.)$.

Given complete markets, aggregation is not needed for the existence of a representative agent. This is true even if individuals are heterogeneous in preferences and wealth, provided that they have the same δ and the same beliefs. Moreover, the representative individual's relative elasticity is no larger than the most risk averse individual and no smaller than the least risk averse individual.

Still with respect to the hypothesis, we follow the didactic Cochrane's (2001) contribution. He argues that we do not have to assume anything on returns distribution and this pricing equation must hold for any asset, stock, bond, option, etc. This fundamental relation holds for any two periods of a multiperiod model and we do not have to assume i.i.d. returns over time. We do not assume that investors have no nonmarketable human capital or no outside sources of income.

Now we need to revisit this standard framework. Given the central issue of this paper we need to introduce borrowing and delinquency decisions into this model of lifecycle consumption-saving-investment.

3.2. Revisiting CCAPM: Model of Lifecycle Consumption-Saving-Investment-Debt-Delinquency

Now assume that investor still lives infinitive periods, receives stochastic endowments and has free portfolio formation.

Moreover, assume that an investor wishing to borrow in t may issue one-period unsecured debt with a non-state-contingent face value given by b_t. Households issue all unsecured debt to a single lender given a finance rate r_t. This is a usual procedure of addressing credit in a general equilibrium framework.

Now we need to introduce delinquent behavior into this quantitative model of lifecycle consumption-saving-investment-debt.

In the following period, the household has two options: repaying debts as promised, $\boldsymbol{d}_{t+1} = 0$, or he can simply not repay the debt as promised, $0 < \boldsymbol{d}_{t+1} \leq 1$, without seeking formal bankrupt protection. This second option is called delinquency, a kind of partial or even total default, given by $\boldsymbol{d}_{t+1} = 1$.

The financial system imposes a punishment against default behavior. The delinquent household faces endowment garnishment in $t+1$ given by $\boldsymbol{e}_{t+1}.\lambda.\boldsymbol{d}_{t+1}$. In other words, the percentage of endowment available is a linear function of the default decision given by $(1-\lambda.\boldsymbol{d}_{t+1})$.

After this punishment measured by the parameter λ, but still in the same period, in $t+1$, he can roll the debt $b_t.(1+r_t).\boldsymbol{d}_{t+1}$ under the same conditions in which he grants a new residual debt, given by $\boldsymbol{b}'_{t+1}$. In other words, in $t+1$ household decides a total debt level, given by

$$\boldsymbol{b}_{t+1} = b_t.(1+r_t).\boldsymbol{d}_{t+1} + \boldsymbol{b}'_{t+1} \tag{5}$$

In this more realist and less restrictive context, we must replace original budget restrictions (2) and (3) for

$$C_t = e_t - \boldsymbol{p}_t\boldsymbol{\xi}_t + b_t \tag{6}$$

$$\boldsymbol{C}_{t+j} = \boldsymbol{e}_{t+j}(1-\lambda.\boldsymbol{d}_{t+1}) + \boldsymbol{X}_{t+j}\boldsymbol{\xi}_{t+j-1} - b_t.(1+r_t).(1-\boldsymbol{d}_{t+1}) + \boldsymbol{b}'_{t+1} - \boldsymbol{p}_{t+j}\boldsymbol{\xi}_{t+j}, j = 1,\ldots,\infty \tag{7}$$

Substituting (5) in (7), after rolling the debt, one can rewrite future budget constraint as

$$\boldsymbol{C}_{t+j} = \boldsymbol{e}_{t+j}(1-\lambda.\boldsymbol{d}_{t+1}) + \boldsymbol{X}_{t+j}\boldsymbol{\xi}_{t+j-1} - b_t.(1+r_t) + \boldsymbol{b}_{t+1} - \boldsymbol{p}_{t+j}\boldsymbol{\xi}_{t+j}, j = 1,\ldots,\infty \tag{8}$$

Once more, substituting the constraints into the objective function and setting the derivative with respect to b_t and $\boldsymbol{d}_{t+1}$ equal to zero, we obtain the following respective first-order conditions for an optimal consumption-saving-portfolio-debt-delinquency choice,

$$1 = \mathbb{E}_t\left[e^{-\delta}\frac{u'(C_{t+1})}{u'(C_t)}(1+r_t).(1-\boldsymbol{d}_{t+1})\right] \tag{9}$$

$$0 = \mathbb{E}_t[e^{-\delta}u'(\boldsymbol{C}_{t+1})(b_t.(1+r_t) - \lambda.\boldsymbol{e}_{t+j})] \tag{10}$$

Based on this extended set of household choices, we propose estimating the system of Euler equations given by fundamental asset pricing equation (4) for each one among N assets, which remains unchanged, besides first-order conditions (9) and (10). It does not involve complexity and may be useful to deal with asset pricing puzzles.

First-order condition (9) applies to each individual investor and for the unique debt option to which he has access. Comparing it with condition (4), here we need an adjusted (by non-delinquency) stochastic discount factor to price real gross rate on the consumer loan.

Finally, we must highlight the contingent nature of such decisions. Issuing in t one-period unsecured debt is non-state-contingent while in $t+1$ he faces the possibility of a state-contingent default. In this sense, we claim that given this property it works as a versatile asset, which can be useful to span or synthetize contingent claims and thus completing markets.

4. Empirical Exercises

4.1. Revisiting Domestic Market Stylized Facts

Kocherlakota (1996) assess theoretical and empirical attempts to address stock market premium in U.S. The author argues that while there are some plausible explanations for the low level of short-term Treasury Bill returns, the large equity premium is still a puzzle to finance. In this context, there are two relevant tests that can be performed in a GMM setup. Following the related literature, first we use the canonical procedure by testing the conditional moment restrictions associated with (4) aiming to price jointly real gross return on S&P 500 and 90-day Treasury Bill in U.S. For our purposes to evaluate our extended system of first-order conditions, in the second test, we just add moment restrictions (9) and (10). This pricing test procedure enables us to infer about the absolute performance of the extended system of moment conditions. It also allows us to compare our main results with those found using the canonical system. These results are reported in Table 2.

Reassured that our system of Euler equations does a good job in accounting for equity premium, for reasonable preference parameters and overall fitting, we proceed to a second exercise. To account for the cross-section behavior of domestic assets, we also perform pricing tests for the six Fama and French (1993) benchmark portfolios: dynamic portfolios extracted from the Fama–French library. These results are reported in Table 3.

For both exercises, we use two preferences. Their specifications are described in the following subsection.

4.2. Preferences Specification

Under a consumption-based approach, it is usual to estimate the utility function parameters and then to test the associated system of orthogonality restrictions using GMM. We use as a benchmark the standard model of consumer preferences (CRRA), which is explored empirically by the majority of articles about elasticity in consumption. This preference it is scale-invariant and if agents have different levels of wealth but the same utility, then this will also be the utility of the representative agent. Although it has been extensively used in finance due to its empirical, analytical and intuitive convenience, it really does not work well in practice, there being evidence of its incapability to account for stylized facts. The SDF of this utility (M_{t+1}^{CRRA}) is given by

$$M_{t+1}^{CRRA} = e^{-\delta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}, \text{if } \psi \neq 1 \text{ and } M_{t+1}^{CRRA} = e^{-\delta}\frac{C_t}{C_{t+1}}, \text{if } \psi = 1 \tag{11}$$

The main disadvantage of (11) is that the Arrow-Pratt measure of relative risk aversion, γ, is the inverse of the agents' elasticity of intertemporal substitution in consumption parameter, ψ. Thus, allowing for high risk aversion to account for the equity premium implies to accept a very low intertemporal marginal rate of substitution. As a consequence, whenever one is willing to accept the high coefficient of relative risk aversion that is needed to correctly price the equity premium, the risk-free puzzle arises. To avoid this trap Epstein and Zin (1991) define more general preferences which preserve many of the attractive features of power utility as the scale-invariance and disentangle both parameters. The SDF of this utility (M_{t+1}^{EZIN}) is given by

$$M_{t+1}^{EZIN} = e^{-\delta\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}\boldsymbol{R}_{w,t+1}^{\theta-1} \tag{12}$$

where $\theta = \frac{1-\gamma}{1-\psi^{-1}}$ and $\boldsymbol{R}_{w,t+1}$ denotes the return on the wealth portfolio.

4.3. Data and summary statistics

In terms of sample size, our main limitation for the time-series span used here regards the credit variables; delinquency rate on consumer loans are available only since 1987, for instance. The largest sample covers the period from 1978:1 to 2018:1, at a quarterly frequency, comprising 125 observations. Macroeconomic and financial variables were extracted from Federal Reserve Economic Data (FRED). Except for the delinquency rate all nominal variables were transformed into real using the corresponding U.S. Consumer Price Index.

Table 1 contains some summary statistics for all exogenous and endogenous variables.

Table 1 Summary statistics [a]

	Mean	St. dev.
Exogenous variables - Assets		
Real Return on 3-Month Treasury Constant Maturity (% per quarter)	0.029%	0.523%
Real Return on S&P 500 (% per quarter)	1.294%	6.721%
Real Return on Fama/French portfolio #1 (% per quarter)	2.528%	12.386%
Real Return on Fama/French portfolio #2 (% per quarter)	3.566%	9.723%
Real Return on Fama/French portfolio #3 (% per quarter)	3.797%	10.654%
Real Return on Fama/French portfolio #4 (% per quarter)	3.075%	8.352%
Real Return on Fama/French portfolio #5 (% per quarter)	2.817%	7.816%
Real Return on Fama/French portfolio #6 (% per quarter)	3.034%	9.365%
Other exogenous variables		
Real Return on Stock market capitalization Wilshire - wealth proxy (% per quarter)	1.786%	7.891%
Real Finance Rate on Personal Loans at Commercial Banks (% per quarter)	2.218%	0.402%
Real Percapita Seasonally Adjusted Personal Income (Thousands of 2017:4 Dollars)	$ 43.33	$ 4.96
Investment to Capital Stock Ratio (%)	6.113%	0.583%
Dividend to Price Ratio (%)	2.189%	0.667%
Endogenous variables		
Delinquency Rate on Consumer Loans, All Commercial Banks (%)	3.253%	0.654%
Real Percapita Seasonally Adjusted Consumer Loans at All Commercial Banks (Thousands of 2017:4 Dollars)	$10.02	$1.38
Real Percapita Seasonally Adjusted Personal Consumption Expenditures: Nondurable Goods and Services (Thousands of 2017:4 Dollars)	$31.87	$3.23

[a] Quarterly series from 1987:1 to 2018:1 (125 observations). Data source: FRED.

The average risk premium on S&P 500 is 5.2% at an annual rate, a little lower than 6% observed in annual U.S. data from 1889–1978 originally studied by Mehra and Prescott (1985) and Kocherlakota (1996). As usual, the correlation of per capita consumption growth with stock returns (0.11) is only slightly bigger than its correlation with bond returns (-0.05). However, the correlation of per capita consumption growth with the real gross rate on personal loan discounted by delinquency is higher, 0.35. Corroborating Bordo and Jeanne's (2002) pass-through we find a positive correlation between loan and real return on the stock market proxy, 0.07. Still according to previous statistics reported in this literature, all six Fama/French dynamic portfolios formed on size and book-to-market show higher values of mean and volatility than S&P500.

We emphasize the order of magnitude of the volatility of loan and delinquency endogenous variables, a crucial element in our extended household decisions framework, characterized by first-order conditions (9) and (10).

4.4. GMM Estimation Setup and Instruments

In choosing how to weight the moments used for estimation, we face a trade-off between attaining full efficiency and correctly specifying the structure of the variance–covariance matrix. To implement a test to account for the EPP based on the canonical system of restrictions and on the extended system given by (4), (9) and (10), we will use optimal GMM, because we have a reasonable number of moment conditions vis-à-vis our sample size.

Our estimates are produced by a procedure which performs a single weight iteration after the initial 2SLS estimates and then iterates coefficients to convergence. We use HAC (Newey-West) as the weighting matrix which is a heteroskedasticity and autocorrelation consistent estimator of the long-run covariance matrix.

As instruments, we use specific financial variables carefully chosen according to their forecasting potential.

For the EPP, we follow Hansen and Singleton (1982) by using consumption growth and other endogenous and/or exogenous variables associated with the respective preference lagged one period. Considering the fact that expected returns and business cycles are correlated, we use stronger instruments as a robustness check. We use macroeconomic variables with documented forecasting ability: Dividend-Price ratio (DP) and Investment to Capital Stock ratio (IC), following Cochrane's (1996) procedure. Aiming to price Fama-French portfolios, we use endogenous and/or exogenous variables associated with the respective preference lagged one period.

4.4. Equity Premium Puzzle: Pricing Test Results

For our purposes, there are two relevant tests that can be performed in a GMM setup.

The first is a test of whether the parameters are statistically zero, which uses a robust Wald test on the individual and joint significance of them in these systems. The second is a standard overidentifying restrictions test (J-test). It has the usual interpretation of a test of orthogonality between the errors in each moment restriction and the instruments used in GMM estimation, thus being a specification test for the validity of instruments.

Table 2 displays the results of consumption-based kernels in pricing returns related to EPP.

When the model omits household decisions on loan and delinquency, individual parameters δ and ψ are statistically zero, at the 5% significance level, for both preferences specifications based on all four instrument sets used. Only at the 10% significance level, we are able to find some non-zero values for ψ ranging from 0.21 to 0.71. The estimation of θ in Epstein and Zin (1991) preferences are significant at 1% significance, except for the first instrument set. Also, Wald-test results show almost no sign of joint significance for CRRA model, while all parameters seem to be jointly statistically non-zero at 1% level in Epstein and Zin preferences.

In the right side of Table 2 we present the results considering all household primitive decisions, including loan and delinquency. To summarize, all parameters in both preferences frameworks using any of the instrument sets are individual and jointly significant at 1% level, except for δ, non-zero only at 5% in Epstein and Zin model with stronger instruments. Regarding J test, we cannot reject the null of the suitability of the model. Moreover, the values of parameters seem to be closer to the values expected by theory and by calibration exercises.

Concerning the elasticity of intertemporal substitution, we follow Lucas' (1990) rule of thumb which relates interest rates r_f with a consumer's subjective time discount rate and consumption growth via

$$r_{f,t} = \delta + \psi^{-1}. \ln(\boldsymbol{C}_{t+1}/C_t) \tag{13}$$

Based on the average value of the range of our estimates of δ and the time series from 1987:1 to 2018:1, we should rule out an elasticity of intertemporal substitution below 0.31. Our estimates of ψ range from 0.43 to 0.55, except for Epstein and Zin specification with the last instrument set used. We find significant values for Arrow-Pratt measure of relative risk aversion ranging from 1.88 to 3.51 when the household also decide on borrow and default.

A classical issue in this literature is forecasting risk-free rate given by the inverse of expected SDF. Comparing both systems of restrictions and both preferences specifications, we find the lower level of mean square error when we use our extended system of Euler equations and the consumer has a CRRA preference. This error is 0.047%, while the errors are 0.365% and 0.443% when we use canonical system with CRRA and Epstein-Zin preferences, respectively. Comparing with previous evidence, Hansen and Singleton (1982) find ψ close to unity. This study follows the empirical related literature by including once lagged consumption growth among instruments which, according to Hall (1988), would lead to endogeneity and consequently high ψ estimates. We deal with Hall's criticize by circumventing endogeneity problem, i.e. we rely on exogenous instruments. However, while ψ estimates in Hall's study are insignificant avoiding endogeneity, we strongly reject individual and jointly significance of ψ when household also decide about loan and delinquency, given all instrument sets used here.

Table 2 Equity Premium Puzzle: Estimating nonlinear Euler system of equations [a, b, c]

	Household decisions: consumption, saving and investments				Household decisions: consumption, saving, investments, credit and delinquency			
	Conditional restrictions (4)				Conditional restrictions (4), (9) and (10)			
CRRA specification - equation (11)								
Parameters	Inst. Set 1	Inst. Set 2	Inst. Set 3	Inst. Set 4	Inst. Set 1	Inst. Set 2	Inst. Set 3	Inst. Set 4
δ	0.00038	-0.00888	0.00233	-0.00609	-0.00505 **	-0.00574 **	-0.00434 **	-0.00572 **
	[0.8988]	[0.2989]	[0.3437]	[0.0922]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
ψ	1.13006	0.21469	0.77594	0.47147	0.54972 **	0.43675 **	0.52375 **	0.49926 **
	[0.3559]	[0.0719]	[0.1115]	[0.0720]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
λ					0.23539 **	0.23651 **	0.23713 **	0.23802 **
					[0.0000]	[0.0000]	[0.0000]	[0.0000]
Overall fit (J-statistic)	0.02710	0.04146	0.05823	0.05261	0.18434	0.18620	0.20007	0.15198
	[0.9865]	[0.9998]	[0.9999]	[0.9997]	[1.0000]	[1.0000]	[1.0000]	[1.0000]
Wald test - H0: δ=0, ψ=0, λ=0	1.26269	54.01565 **	2.63503	83.78310	7657.802 **	28199.23 **	256096.3 **	14441.13 **
	[0.5319]	[0.0000]	[0.2678]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
Epstein-Zin specification - equation (12)								
Parameters	Inst. Set 1	Inst. Set 2	Inst. Set 3	Inst. Set 4	Inst. Set 1	Inst. Set 2	Inst. Set 3	Inst. Set 4
δ	0.01872	-0.00305	0.00187	-0.01037	-0.01615 **	-0.00591 **	-0.00658 **	-0.02027 *
	[0.2454]	[0.5604]	[0.5076]	[0.7114]	[0.0000]	[0.0000]	[0.0000]	[0.0396]
ψ	-0.17864	0.37065	0.71056	0.43324	0.42977 **	0.51476 **	0.47694 **	0.23829 **
	[0.2419]	[0.0815]	[0.0931]	[0.7444]	[0.0000]	[0.0000]	[0.0000]	[0.0004]
λ					0.23632 **	0.23539 **	0.23699 **	0.24013 **
					[0.0000]	[0.0000]	[0.0000]	[0.0000]
θ	-2.19969	0.97436 **	0.96690 **	0.44950 **	0.82493 **	0.93520 **	0.89827 **	0.78535 **
	[0.2624]	[0.0000]	[0.0000]	[0.2670]	[0.0000]	[0.0000]	[0.0000]	[0.0004]
Overall fit (J-statistic)	0.00524	0.04054	0.05297	0.04918	0.18053	0.19034	0.19802	0.15300
	[0.9423]	[0.9999]	[1.0000]	[0.9971]	[1.0000]	[1.0000]	[1.0000]	[1.0000]
Wald test - H0: δ=0, ψ=0, λ=0, θ=0	103.6811 **	661.6879 **	626.2023 **	29.78637 **	11586.44 **	28029.62 **	300831.8 **	7778.105 **
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]

[a] Hansen's (1982) efficient GMM (update weights once and then iterate coefficients to convergence) used to test Euler equations and to estimate parameters over the period from 1987:1 to 2018:1 (125 observations). [b] Instrument sets 1 to 3 are comprised by variables associated with the model in question: 1) endogenous lagged one period; 2) exogenous variables lagged one period; 3) endogenous and exogenous variables lagged one period. Fourth instrument set is composed by IC and DP lagged on period. [c] Respective p-values are reported in the box brackets. Data source: FRED. * p-value <0.05. ** p-value <0.01.

4.5. Discussion on Credit Market

According to results reported in Table 2, the parameter λ, related to the garnishment of a delinquent household endowment, ranges from 0.23 to 0.24, while in Athreya (2012) this parameter is assumed to be lower than 0.1. An interesting implication of this parameter is due to $\partial b_t/\partial\lambda > 0$, i.e. the possibility to raise household credit grant given a higher value for the garnishment of endowment parameter, without changing loan rates.

Our extended version of households' primary decisions enables us to infer on the locus (Figure 2) of possible combinations of delinquency (horizontal axis) and real per capita consumer loans (vertical axis) given our estimates using Epstein and Zin preferences and the fourth stronger instrument set, for instance. More specifically, we plot three set of possible ordered pairs of delinquency and loan given unconditional moments related to restrictions (9) and (10). The difference is only the endowment level. This approach can be useful to draw bounds and then better understanding the dispersion of loan volume over time, which seems to be higher than our model predicts. In other words, we should theoretically observe smoother household loan cycles, mainly in scenarios with low delinquency and high level of loans.

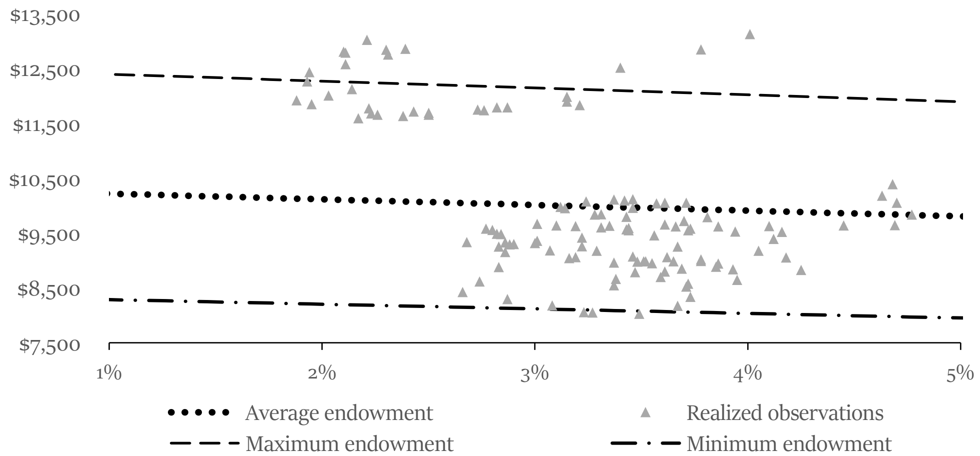

Fig. 2. Delinquency (%) vs Loan (Thousands of 2017:4 Dollars) from 1987:1 to 2018:1.

4.6. Additional Pricing Test Results: Dynamic Portfolios

Table 3 reports the main results of pricing test of Fama/French dynamic portfolios.

Table 3 Revisiting other domestic market stylized facts [a, b, c]

	Household decisions: consumption, saving, investments, credit and delinquency						
	Conditional restrictions (4), (9) and (10)						
CRRA specification - equation (11)				Epstein-Zin specification - equation (12)			
Parameters	Inst. Set 1	Inst. Set 2	Inst. Set 3	Parameters	Inst. Set 1	Inst. Set 2	Inst. Set 3
δ	0.01829 **	0.02075 **	0.02079 **	δ	0.44367	0.03015 **	0.03000 **
	[0.0000]	[0.0000]	[0.0000]		[0.9119]	[0.0000]	[0.0000]
ψ	0.70683 **	0.75013 **	0.68138 **	ψ	0.00422	0.18191 **	0.15548 **
	[0.0000]	[0.0000]	[0.0000]		[0.9160]	[0.0000]	[0.0000]
λ	0.23539 **	0.23613 **	0.23620 **	λ	0.23561 **	0.23579 **	0.23592 **
	[0.0000]	[0.0000]	[0.0000]		[0.0000]	[0.0000]	[0.0000]
				θ	0.00667	0.25150 **	0.23698 **
					[0.9159]	[0.0000]	[0.0000]
Overall fit (J-statistic)	0.19139	0.20336	0.20495	Overall fit (J-statistic)	0.18750	0.20402	0.20537
	[1.0000]	[1.0000]	[1.0000]		[1.0000]	[1.0000]	[1.0000]
Wald test - H0: δ=0, ψ=0, λ=0	14730.07 **	151660.1 **	10657241.0 **	Wald test - H0: δ=0, ψ=0, λ=0, θ=0	19995.74 **	123880.5 **	1626566.0 **
	[0.0000]	[0.0000]	[0.0000]		[0.0000]	[0.0000]	[0.0000]

[a] Hansen's (1982) efficient GMM (update weights once and then iterate coefficients to convergence) used to test Euler equations and to estimate parameters over the period from 1987:1 to 2018:1 (125 observations). [b] Instrument sets 1 to 3 are comprised by variables associated with the model in question: 1) endogenous lagged one period; 2) exogenous variables lagged one period; 3) endogenous and exogenous variables lagged one period. [c] Respective p-values are reported in the box brackets. Data source: FRED. * p-value <0.05. ** p-value <0.01.

In this robustness exercise, we reject at 1% level the null that parameters are individual zero significant for both preferences, except for Epstein and Zin model with endogenous instruments. We reject the null of joint insignificancy for all cases. With regards the overall fitting, we cannot reject the null of the suitability of the model. We find that elasticity ranges from 0.15 to 0.75, while risk aversion ranges from 1.33 to 2.29. The values of the parameter related to the garnishment of a delinquent household endowment remain similar to those previously reported. Our results suggest that we are able to account for cross-section dynamics of domestic assets in U.S.

5. Concluding Remarks

We address a central issue in consumption-based asset pricing literature by introducing debt and delinquency as endogenous choices of the investor. As a consequence, we propose adding two investor's first-order conditions to the usual system of orthogonality restrictions. Theoretically, the possibility of issuing a debt and in the next period skip or not this obligation can be seen as an additional asset relevant in terms of market completeness. In practice, we are imposing that the researcher necessarily has to account for this asset, in addition to the assets that he wishes to price. Our satisfactory empirical findings come from this spam of contingent claims, rather than higher variance of the stochastic discount factor, which has been usually the purpose in many other approaches.

Estimating additional Euler equations does not involve higher complexity. Also, it exempts us from making additional assumptions beyond what is needed to test the moment conditions that characterize tests in levels. Our empirical findings suggest that this extension of the household vector of decisions seems to be useful to account for Equity Premium Puzzle in U.S. Moreover, our work provides empirical grounds to believe that our framework is able to she light on other domestic market stylized facts that escape consumption-based models.

Our model is supposed to provide researchers to combine it with any among all these research promising routes. To account for other domestic market anomalies, one can also use returns on the Fama/French industry portfolios or even their benchmark portfolios formed on operating profitability, investment, earning/price, cashflow/price and momentum, for instance. Given the high volatility of credit and the levels of delinquency in emerging economies, our approach can be useful to price correctly the equity premium in these countries. Finally, da Costa et al. (2015) and Matos and da Costa (2016) argue that progresses on preference-based solutions to puzzles in domestic financial markets will also deal with puzzles in the foreign exchange market. So, we invite researchers to assume this wider vector of investor decisions to account for international finance puzzles.

Compliance with Ethical Standards

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Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

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We add to asset pricing theory by introducing household's debt and delinquency decisions into an otherwise standard quantitative model of lifecycle consumption-saving-investment. This extended system of Euler equations incorporates two new first-order conditions. It does not involve higher complexity, does not alter consumption-based fundamental asset pricing equation and exempts us from making additional premises. We perform two empirical exercises, one to account for equity premium in U.S. and other to price six Fama-French dynamic portfolios. We are able to find significant elasticity ranging from 0.15 to 0.75. These additional decisions may be playing a role in terms of completing markets.

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