School Enrollment and Labor Market Performance: Theory and Evidence from Brazil*

Maurício Benegas[†] Márcio Corrêa[‡]

October 26, 2016

Abstract

This paper studies the economic impact of an school enrollment policy in a large developing economy, Brazil. We build and numerically solve a two-sector matching model of the labor market where the skilled segment of the economy is composed of a continuum of workers who differ in the quality of the school attended and firms that endogenously supply vacancies. We show that this policy may or may not generate negative effects on the economy. The basic mechanism behind these results is the composition effect. It mainly results from individual self-selection in education, as previous advocated in literature. However, the composition effect in the present model is due to the school quality distribution and the link between public school enrollment policy and the demand of education. We shown that the more sensitive is the aggregate demand of education to the school enrollment policy the more likely is the evidence of negative effects on the economy.

KEYWORDS: Search frictions; Human Capital; Public Policy; Composition Effect; Economic Development.
JEL CLASSIFICATION: J24; J31; J38; O11

---

*We thank Tiago Cavalcanti, Fernanda Estevan, Marcelo Silva, Cézar Santos and seminar participants at PIMES-UFPe and XXXVII SBE Annual Meeting, Brazil.

[†]CAEN - PostGraduate Studies in Economics, Universidade Federal do Ceará, Brazil. Email: benegas@caen.ufc.br.

[‡]CAEN - Postgraduate Studies in Economics, Universidade Federal do Ceará, Brazil. Email: marciovcorrea@caen.ufc.br.

1 Introduction

Perhaps one of the most important labor market problems in many developing countries is the evidence of a high and persistent unskilled sector. In Brazil, for instance, although 95% of the total children population are enrolled in primary education, only 55% succeeded to conclude the secondary education[1]. It is also well know the positive impact that education has in human and economic development. Card (1997), for instance, defended that an additional year of schooling is responsible for an increase in the individual wage rage that ranges between 6% to 10%.

There are also no doubts that human capital investments reduce the incidence of social problems, such as crime and health problems, and indirectly improve a country political and economic institutions.

Different arguments have been proposed in the literature to understand the phenomenon of low school enrollment and the high school drop out rate[2]. Becker (1993), for example, defended that the accumulation of human capital is the result of an individual decision that could be compared to other forms of investments. In his view, it is expected to see an overall increase in the school enrollment rate as the expected benefits of this investment exceeds its costs. In this way, the cheaper it is the schooling activity, the higher will be the participation rate.

Becker also defended that policies intended to increase the option value of education or policies designed to reduce the schooling costs, for example, are all expected to influence the workers investment decision.

Another argument was proposed by Eckstein and Wolpin (1999). They defended that the decision to attend school is the result of the difference between individuals' perceived payoff received from the education, which is uncertain, and the utility value related to attending schools. They considered that students begin their educational life with similar preferences, skills and motivations. However, as time moves on, there can be changes in these individual characteristics and the likelihood of school failure can increase, reducing the school performance and the agent perspective to become schooled. They concluded that school drop out can be related to some individual traits, such as lower school ability and motivation, higher labor market return from the unskilled sector and higher costs to attending schools.

Other possible explanation for the widespread incidence of lower levels of human capital investments in developing economies was proposed by Basu and Van (1998).

[1] See UNESCO (2011) on educational figures of other Latin American economies.

[2] According to a survey conducted in 1979 by the National Longitudinal Surveys of Labor Market Experience, and presented in Eckstein and Wolpin (1999), 30 % of the total of US white male youths that decided to drop out school choose this option because they "didn't like the school" whilst 14% decided not to study because they do not have a good job offer.

The authors defended that workers will abandon schools and increase their labor force participation in the unskilled and informal sector as they have difficulties to make ends meet. In this way, the more difficult it is to survive, or the weaker it is the institutional environment designed to ban child labor, the higher will be the student drop out rate.

Using a closer argument, Munshi and Rosenzweig (2006) and Foster and Rosenzweig (2006) defended that the incidence of international differences in school attainment rates can also be attributed to differences in institutional qualities or to a country technological experiences. The argument is that schooling net returns may change in periods of economic depression or due to poor designed institutions, a phenomenon typically present in developing economies.

It has also been defended the existence of a coordination problem between firms and workers that hampers human capital investments. According to this view, firms will always invest in the high skill sector if exist a sufficient supply of skilled workers in the economy. However, since the worker decision to become educated also depends on the labor market expected returns, it is possible to arrive at an equilibrium characterized by a lower skilled sector and a large number of unskilled workers[3].

Two points emerge from all these previous mentioned contributions. First, it should be pointed out that basically all the prior studies predict that as the net benefit to become educated becomes higher than the returns received from the unskilled sector, it is expected to verify an increase in the school enrollment. Second, any educational policy must be designed considering the general equilibrium effects that exists between the educational and the labor market. If these previous link exists and are not taken into account, the impact of a policy designed to increase the size of the skilled sector, for example, can generate unexpected effects in the labor market.

Taking these theoretical and empirical predictions into account many developing countries have implemented educational policy reforms aiming to increase the mass and quality of their labor force. Brazil, for instance, has implemented a set of policies aiming to reverse their worst educational statistics. Mainly led by a 110% increase in expenditure per student enrolled at primary and secondary public schools, between 2002 and 2015, the country has shown an school enrollment rate above the world average. According to OECD (2015) the mass of individuals aged between 25-64 that attained secondary education in Brazil increased from 28% to 61% of the total

[3]Limited quantity or lack of access to schools, low school qualities or credit constraints have also been proposed as the main causes of poor countries lower performance in school attainment. See, for example, Evans and Schwab (1995) and Duflo (2001).

population in thirty years. The share of the population aged between 25 and 64 years old that completed a university degree also increased by 3%, between 2009 and 2013.

However, although Brazil has made progress on increasing school attendance and reducing the school drop out rate, problems related to school quality still persist. The school failure rate in secondary education has increased nearly 82%, between 1990 and 2011 and the Illiteracy rate still persists in the poorest region of Brazil. In northeast region, for instance, the illiteracy rate is around 17% whilst in the south region it is 4,4%.

Another intriguing factor is the low connection between the recent increase in school enrollment and aggregate productivity. As it can be seen in figure 1, the average productivity growth rate in Brazil has not followed the overall increase in school attainment. The years of schooling rose by 163% over the past thirty years in Brazil. It represents an increase well above the South Korea average, which amounted to 48%. However, the productivity gains in Brazil are approximately nil while in the Asian country it grown up by 184%, at the same period.

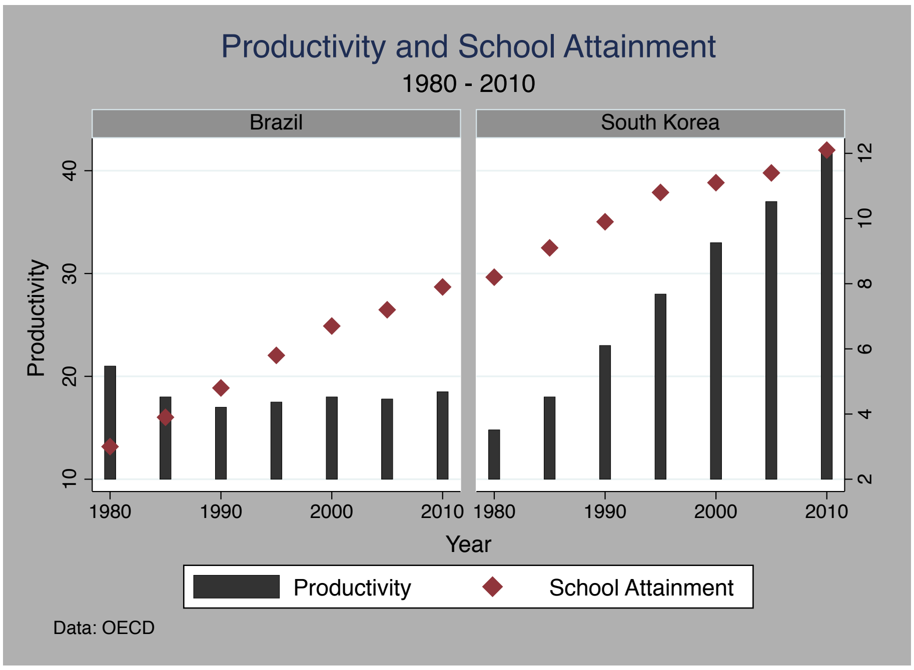

Figure 1: Productivity and School Attainment, Brazil and South Korea

Human capital misallocation, low quality of the local institutions, culture or low investment levels of local firms have all been accused as the main reasons behind the previous relationship. The quality of Brazilian school system and the increased labor demand for unskilled workers are also blamed as responsible for this result.

In this paper, we propose an answer for the lack of connection between schooling enrollment and the low labor force productivity in Brazil. As in the previous literature we argue that there is a link among school quality, investment costs and the relative returns of educational investments that may generate the previous outcome.

In our model, an individual that works on the skilled sector must go to schools for a fixed period of time and the set of school vacancies available is heterogeneous with respect to their quality. School quality is fixed by the government through a policy that allocates the best schools available to the best individuals in the economy.

We shown that a policy designed to increase the school enrollment rate may generate the previous empirical outcome observed in Figure 1. In particular, a school enrollment policy implies a reduction on the average quality of the skilled workforce and an increase on the skilled labor force. This result resembles the composition effect proposed by Charlot and Decreuse (2005a). However, unlike these authors, in our model it is the result of the impact of the school enrollment policy on the aggregate demand and supply of education.

To better understand this result, consider an economy with a low level of school enrollment. The government then puts in place a policy that increases the set of school options available in the economy. Consider, without loss of generality, that this policy is characterized by an increase on the school quality options available to each individual.

The expansion on the school quality provision generates a positive effect on the economy. The skilled sector is directly encouraged by a higher and better skilled labor force. However, the government school supply policy has also an indirect effect on the economy. As there is an increase on the school quality options available to each agent, they become less reluctant to study. Then, there is an increase on the demand of education. This higher labor demand reduces the skilled labor force and depresses the skilled sector. We show that the final effect depends on theses two opposite effects.

To provide a quantitative evaluation of the aggregate impact of this educational policy, we calibrate the model for the Brazilian economy and simulate it for different school enrollment policy parameters. We show that an open enrollment policy implies that the skilled labor force converge to the mean of the school quality distribution available in the economy. We also demonstrate that the composition effect may or may not be active in the economy.

The present model is related to a growing literature that studies the determinants of the workers' investment in education and how the decentralized equilibrium compares to the social optimum outcome. On the one hand, some authors have argued that the educational market decentralized equilibrium generates a low and inefficient level of educational investments. Thus, government interventions are necessary to reduce inefficiency[4]. On the other hand, some authors defend the evidence of a high and inefficient levels of educational investments. Charlot and Decreuse (2005a), for instance, argue that when educational investments and labor market returns are positively related, the phenomenon of overeducation would arise. They argue that as the size of the educated workforce increases, there would be a reduction in the average ability of both the schooled and the unschooled segments of the labor force, which implies a decrease on the firms incentives to open new vacancies. They conclude that any welfare improvement policy should be designed to deter low- skilled individuals from entering the schooled segment of the economy.

As in the present model, Charlot and Decreuse (2010) extended their previous results to a scenario where individuals are additionally heterogeneous with respect to their costs to acquire education. They show that workers' self-selection in education generates the composition effect. Consequently, there is a reduction on the average workforce ability and the job creation dynamics in both economic sectors, as individuals who haven't studied decide to invest in education. They conclude that any efficient policy must be designed to attract highly skilled individuals to the group with high educational costs and those who are less skilled to the group with low educational costs.

In our opinion, the main feature of the present model is to shown that the composition effect can be the result of the government intervention on the educational market. We also show that inefficiency persists in equilibrium and that the public policies may exacerbate this problem.

Besides this introduction, this paper has four more sections. In the next section we present some empirical regularities of the Brazilian schooling and labor market. We introduce the benchmark model and describe the decentralized and the social welfare allocations in the following section. Section 4 quantitatively studies the effects of and enrollment policy experiments in the labor and schooling market equilibrium while the last section contains the main concluding remarks.

[4]Acemoglu (1996) is remarkable example.

2 Empirical Facts: Labor and Public Schooling in Brazil

In this section we use three different surveys to present some empirical regularities of labor and the public educational markets in Brazil. Namely, we use the Brazilian Monthly Employment Survey - Pesquisa Mensal de Emprego, PME - from 2002 to 2015 to present some empirical facts of the Brazilian labor market and the link between labor force participation and educational attainment. We also use data from the 2002 and 2008 Brazilian Household Budget Surveys - Pesquisa de Orçamentos Familiares, POF - from the Brazilian Statistical Institute, to characterize family and individual expenditures on education. Finally, we use the Brazilian Assessment of Education Progress - Prova Brazil - from 2007 to 2013, to collect data and show the evolution of student progress and the overall quality of public education in Brazil.

2.1 Labor Market and Educational Attainment

The Pesquisa Mensal de Emprego is the main employment survey in Brazil. It is a monthly based inquiry conducted in the six main metropolitan areas of Brazil - Belo Horizonte, Porto Alegre, Recife, Rio de Janeiro, Salvador and São Paulo by the Brazilian Statistical Institute - IBGE. It is designed to follow the individual's behavior on the labor market. This is done during an initial period of four consecutive months. In the following eight months, this initial group is eliminated from the sample, only returning after eight months. Two new groups are included in the survey: a new group enters at every four months. In this way, at two completed years, three different groups of individuals are surveyed providing detailed information on individual employment statuses, level of education, wages and other sources of individual and family income, for instance[5].

The sample used in this section includes all months between March 2002 and June 2015. Figures 2 and 3 present the unemployment rate, the average productivity of the economically active population and their trends in Brazil[6]. It can be seen that the unemployment rate reduced significantly, from 12% in early 2002 to something close to 5% at the second quarter of 2015. In turn, real productivity raised by 12%

[5]Unfortunately, the survey was extensively modified in 2002 challenging the reconciliation of data before and after 2002. In 2013, the Brazilian Statistical Institute also decided to change the population weights used in the survey, with new population projections from the 2010 Census. Since there is by now no public information available of the 2002, 2006 and 2011 reweighed surveys, we use the pre reweighting available data from years 2002 to 2015.

[6]Productivity is measured as the real output per employed workers (in millions of reais) from the Brazilian Nacional Accounts - Sistema de Contas Nacionais. The data are quarterly averaged. We use the Hodrick-Prescott filter with smoothing parameter $\lambda = 1.600$ to detrend the data.

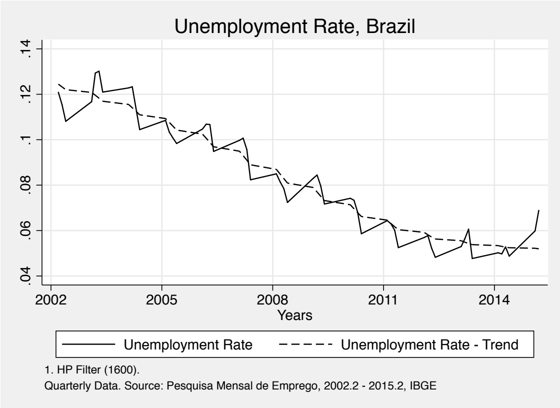

Figure 2: Unemployment Rate

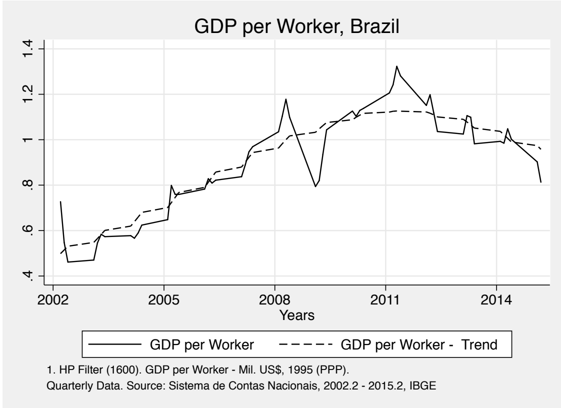

Figure 3: GDP per Worker

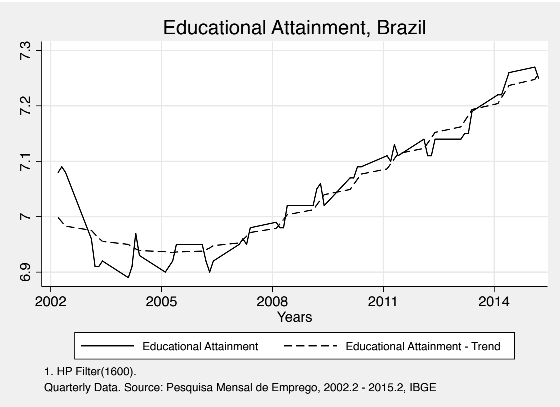

Figure 4: Educational Attainment

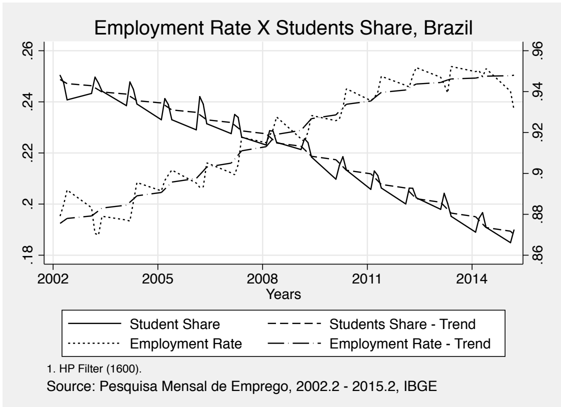

Figure 5: Employment and Students Share

in the same period. The -0.84 correlation coefficient ensures that the two previous series are negative correlated.

The following figure presents the average years of schooling in Brazil. It can be seen that the educational attainment falls from the beginning of 2002 up to the third quarter of 2006. Then, it shows an annual increase of $0,01$ points between the end of 2006 and the second quarter of 2015[7]. In turn, figure 5 presents the proportion of students to the total population and the employment rate in Brazil. It shows that the share of students reduced from 25% of the total population at the first quarter of 2002 to around 20% at the second quarter of 2015. The $-0,94$ correlation coefficient between the share of students and the Brazilian employment rate shows the negative and strong link that exists between these two series.

As in U.S., formal education in Brazil is divided in educational stages. The primary education - Ensino Fundamental - involves nine years of education. It

[7]Although there was a significant increase in years of schooling in Brazil, the distance to the most developed economies persists. According to the OECD Education database, the average of successfully completed years of schooling at the OECD was 11.9 years at the end of 2010.

is compulsory for all children between six and fourteen years old. The secondary education - Ensino Médio - comprises three years of schooling. Before the reform of the educational system in Brazil, the nine years of the primary education were known as: the literacy year and the years 1 to 8, respectively. After the reform, that took place in 1996, these years became known simply as years 1 to 9 of the elementary school. However, this is not the most important change. The focus on teachers' quality and on school management as well as the improvement of the national school curriculum, for instance, came to change the educational policy in Brazil.

Figure 6 reveals that the shift in the number of students at the elementary and the middle school dominates the bulking of agents that left school. The reduction in the number of students enrolled at the primary and secondary education was around 20% and 5% between the years of 2002 and 2014, respectively.

The mass of students enrolled at the higher education increased significantly during the same period. However, it also dropped between 2009 and 2011, following the overall reduction in the school enrollment rates in Brazil.

It is also worth noting that the increase in the mass of students enrolled in higher education is also due to the re-entry of students who previously completed the secondary school and decided to work instead of studying. This can be seen by the ratio of the number of students enrolled at higher education and the number of students enrolled at the secondary school. It jumped from 0,51 in 2002 to 1,03 at the end of 2015. In turn, the ratio of the number of students enrolled in secondary and primary education just increased by 20% at the same period.

The following figure presents the school dropout rate per year of schooling in Brazil. For building this figure we run a Probit regression of the individual decision to work ($Y = 1$) and dummy variables related to the highest completed grade in school:

$$Y = \beta_0 + \sum_i \sum_j \beta_{i,j} D_i^{ES} D_j^G + \epsilon, \quad i \in \{p, s, h\}, \quad j \in \{2, 3, 4, 5, 6, 7, 8, 9\};$$

where D_i^{ES} is a dummy variable related to the educational stage - primary, secondary or higher education - and D_j^G is the school grade dummy variable[8].

Although some caution is needed with the previous regression due to the small sample size, it shows that there is a spike in the dropout rate at the last year of the

[8]Notice, for instance, that $D_P^{ES} = 1$ if the individual is enrolled at the primary education and $D_P^{ES} = 0$, if not. In turn, $D_1^G = 1$ if the agent is enrolled at the first grade and $D_1^G = 0$ if not. The other dummy variables construction follows the same reasoning.

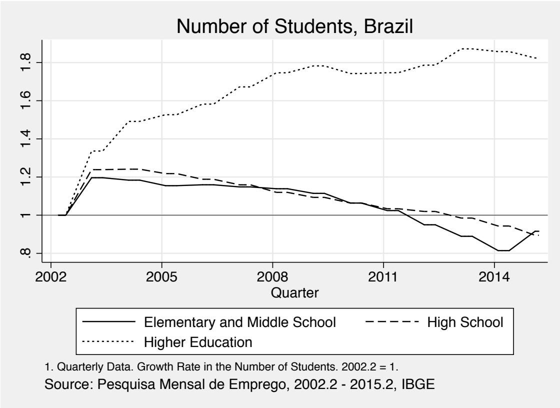

Figure 6: Change in the Number of Students

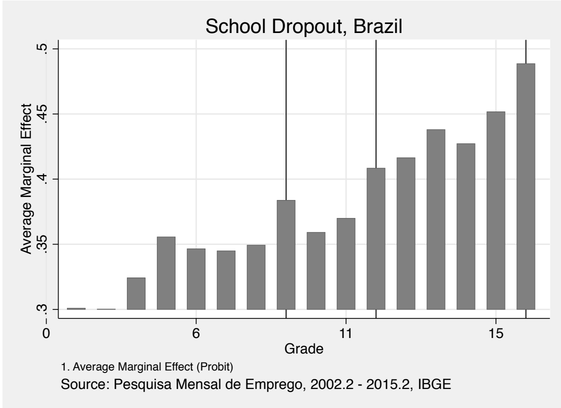

Figure 7: School Dropout

primary education[9]. This can be seen by the increase in the individual probability of leaving school and joining the labor force at year 9. There is also an increase in the dropout rate at the end of the secondary stage of education. Notice that the dropout rate from higher education grows over all the years of the college. The only exception is the third year that presents a small reduction at the individual probability of leaving the higher education.

2.2 Expenditures and the Quality of Public Education

The Pesquisa de Orçamentos Familiares, POF is a survey that aims to characterize households and residents behavior, with a special focus on their budgets. It is usually applied in the period ranging from six to seven years by the Brazilian Statistical Institute and it is mostly designed to describe individual and family endowments, as well as personal and aggregate expenditures incurred in the period ranging from a week to a year. Each household is followed by a period of twelve consecutive months.

The most recent surveys in Brazil are the 2002/03 and the 2008/09. They provide detailed information on the individual annual spending on education, specifying the type and the amount of expenditure. The last available survey covered 55.970 households and 190.159 individuals at the whole country. In turn, the 2002/03 survey includes 48.568 households of an average size of 3,6 members.

The advantage of using POF to obtain detailed data on the individual investments on education is that it is possible to exclude from the sample those individuals that attend private schools and fully describe family income and the total amount

[9]We use the designation of the primary education used in Brazil after the 1996 educational reform. In this way, the year 8 corresponds to the year 7 of the old system. In turn, year 11 corresponds the second year of secondary education at the old system.

Grade	Ratio	Unemployment Rate (%)
Primary School		
year 2	0.020	8,51
year 3	0.021	8,93
year 4	0.024	8,44
year 5	0.030	7,11
year 6	0.022	8,81
year 7	0.023	9,08
year 8	0.017	9,34
year 9	0.048	7,35
Secondary School		
year 1	0.014	10,89
year 2	0.023	10,24
year 3	0.028	9,45

Table 1: Ratio of Expenditure on Education to the Total Income, 2002/03 and 2008/2009, IBGE

of the family expenditures on public education[10].

Table 1 presents the unemployment rate per completed year of education and the ratio of the monthly expenditure on education to the total family income for the six main metropolitan areas of Brazil - Belo Horizonte, Porto Alegre, Recife, Rio de Janeiro, Salvador and São Paulo.

Families expend on average $2,5\%$ of their total income on educational consumption goods. A -0.69 correlation coefficient between unemployment rate and expenditures ensures that the two variables move in opposite directions. The higher the mass of unemployed individuals in a particular household, for instance, the lower the family income and the amount of resources invested in education.

Table 2, in turn, shows that 64% of individuals aged between 13 and 18 years old were composed by students enrolled at public schools, at 2002. The northeast region of Brazil is one of the poorest regions. It can be seen that around 90% of all individuals at the school age attend public schools at this region. At the Southeast, it is around 31%. Households spend 14% of their total household expenditures on education on school commuting costs. The families expenditure on additional school supplies accounted for 14,75% of the total amount.

The number of students enrolled at the public education increased at 2008, with the Southeast leading the increase. There is also a significant reduction on the share of the transportation costs over the total expenditures on education between 2002

[10]Notice that there is no direct costs involved in public education in Brazil. However, public school enrollment involves significative indirect costs that are usually not supported by the Government, as for example the cost of commuting to school, school supplies and in some case school uniform. In this section, we present data on this indirect cost of education.

	%
2002/2003	
Students (13 - 18)	
Public Education	64,24
Northeast	90,57
Southeast	31,57
Expenditures (13 - 18)	
School Supplies	14,75
School Commuting Costs	14,00
2008/2009	
Students (13 - 18)	
Public Education	85,61
Northeast	85,80
Southeast	85,29
Expenditures (13 - 18)	
School Supplies	17,51
School Commuting Costs	7,82

Table 2: Students Enrolled at Public Schools, 2002/03 and 2008/2009, IBGE

and 2008.

The Prova Brazil is a survey that aims to evaluate students performance and assess the quality of the Brazilian public educational system. As the PISA educational evaluation program, it uses standarized tests to evaluate the performance of students enrolled at primary education.

The survey assesses students of 5th and 9th years of elementary education in all public schools with more than 20 students at the urban and rural areas in Brazil on the subjects of mathematics and portuguese - the native language. The grades at the portuguese and the mathematics exams ranges from 0 to 350 and from 0 to 375 for the 5th year of the primary education, respectively. The respective grades for the 9th year ranges from 200 to 400, for the portuguese exam, and from 200 to 425, for mathematics. The school participation at the Prova Brazil is voluntary and students of the 5th year must answer 22 questions of portuguese and mathematics. The 9th year exam is composed of 26 questions on each subject.

Table 3 provides descriptive statistics of the average performance of students enrolled at public schools at the six main metropolitan areas of Brazil - Belo Horizonte, Porto Alegre, Recife, Rio de Janeiro, Salvador and São Paulo. It shows that students enrolled at 9th year of the primary education go slightly better than their counterpart at the 5th year. They correctly answered around 60% of the two exams. It can also be noted that the ratio of the top 10% grades to the lower 10% grades shows a wide dispersion on students performance[11].

[11] This result remains when comparing school performance among the six mains cities of Brazil.

	5th	9th	Total
Mean	196,13	242,32	216,96
Std.Dev.	42,83	44,06	49,09
P_{10}	142,34	182,72	153,39
P_{90}	253,96	301,00	281,07
P_{90}/P_{10}	1,78	1,65	1,83
Maximum	317,17	366,15	366,15
Minimum	94,48	130,43	94,48
Correlation - 5th Grade			
Educational Attainment			0,89
GDP per Worker			0,85
Unemployment Rate			- 0,83

Table 3: Prova Brasil and SAEB, 2007 - 2013, INEP

The correlation coefficients that follows on Table 3 suggest that the score obtained by the students of the fifth year is positively related to the completed years of schooling and the gdp per worker. In turn, the higher the student score, the lower the unemployment rate.

3 The Model

The previous section shown that there exist a strong link among expenditures on education, school quality and the families decisions to invest in human capital accumulation. It has also been shown that the public school quality is low in average terms and that it is unequally provided by the public sector. In this section we propose a model economy that intends to replicate these previous empirical regularities.

The economy is composed of a government that maintains an educational system; a constant population of measure one of individuals and a great number of firms, which once matched with workers give way to a production of a single consumption good whose price is normalized to one.

Firms and workers are risk-neutral and discount the future at the exogenous and constant rate ρ. Let the time be continuous and consider that each firm has access to a production technology that exhibits decreasing returns to scale with labor as the only input.

There are two sectors in the economy: skilled (S) and unskilled (N). Before opening a vacancy, each company must decide in which sector they will produce. Consider, as Smith (1999) and Cahuc and Wasmer (2001), that the size of the labor force employed by each firm is endogenous.

In Recife, for instance, the average score was 180 points whilst at Rio de Janeiro, it was 207 points.

The public educational system is considered, without loss of generality, to be monopolistic in the production of human capital. Let's assume it is composed of a continuum of heterogeneous schools with regards to their quality, q, defined by a distribution $G(q)$ in the interval $[q_L, q_H]$. Schooling is not compulsory.

There is a measure one of infinitely-lived individuals in the economy. They are all born with the unskilled labor market productivity q_L, an individual schooling cost I and the ability to work on the unskilled sector of the economy. However, agents can study at the cost I and become attested to work in the skilled sector[12]. Let $H(I)$, with support in the interval $[I_L, I_H]$, be the distribution of the individual schooling costs[13].

The government observes the individual cost of study. Then, it provides to each agent school offers on the set of school qualities, $[q_L, Q_o(I)]$, where $Q_o(I)$ is a function of the schooling cost I and describes the best school quality option available to agent I[14]. Let the allocation of students to schools be defined by a policy that matches students with costs $I \in [I_L, I_H]$ to schools with qualities $q \in [q_L, q_H]$. Consider that the government assigns the best schools options to the best students and the worst school qualities to the worst group of students in the economy[15].

Notice that in the early stages of their lives, individuals can be studying for an exogenous fixed period of time T in one of the assigned schools. They can also be working or searching for a job placement if they decide not to study. In the remaining periods of their lives, they can only be working or unemployed and searching for a job. Consider, without loss of generality, that the labor market productivity of a skilled individual depends on the quality of the school attended, q.

Let $Q_d(I)$ represents the reservation school quality that leaves an individual with schooling cost I indifferent between entering the labor force as unskilled worker and studying. Agents evaluate working and educational options according to their schooling cost and the labor market returns from their human capital investments in a school of quality q. Whenever $q \geq Q_d(I)$, the individual I decides to go to school,

[12]Considers, as Burdett and Smith (2002), that the two sectors are segmented.

[13]The heterogeneity of schooling costs could be seen, for example, as different direct and indirect costs of study, as proposed by Becker (1962); different family or community endowments, as suggested by Fernandez and Rogerson (1996) and Dottori, Estevan, and Shen (2013) or as different students abilities, as defended by Charlot and Decreuse (2005b).

[14]Since individuals are heterogenous with respect to the schooling cost we will consider the agent I as the individual with schooling cost I.

[15]Notice that we are considering the worst students as the group with higher schooling costs and best schools as the group with best qualities in the economy. We could have defined the school allocation mechanism in the opposite way, that is, the worst schools options to the best workers and the best schools to the worst workers. The basic model results would be the same. The only difference would be that we would have a positive assortative matching between workers and schools instead of a negative one.

thus becoming educated. On the other hand, if $q < Q_d(I)$, this agent decides to work in the unskilled sector, since the labor market returns are bigger than the net benefits received from schooling investments.

Consider, as it is standard in the search literature, that before starting production, workers and firms are involved in a search process to find a productive partner, where k_S and k_N ($k_S > k_N$) represent the search costs of a firm that decides to open a vacancy in the skilled and the unskilled sector, respectively.

The number of job matches formed per period is given by a non-negative, concave and homogeneous degree one matching function, $m(v_i, u_i)$, which is increasing in its arguments. Let v_i represent the vacancy rate and u_i denote the fraction of type $i = \{S, N\}$ unemployed workers in the economy. Through the homogeneity assumption, it can be show that the probability rate of filling a vacancy is given by: $p(\theta_i) = \frac{m(v_i,u_i)}{v_i}$, where $\theta_i = \frac{v_i}{u_i}$ denotes the tightness of the sector i. In turn, the rate at which an unemployed worker moves into employment status is given by $z(\theta_i) = \theta_i p(\theta_i) = \frac{m(v_i,u_i)}{u_i}$.

3.1 Labor Market

3.1.1 Firms

Production can be performed by firms in the skilled and unskilled sectors. The following Hamilton-Jacobi-Bellman equations describe the problem of a representative firm in each sector:

$$\rho\Pi_N(l_N) = \max_{v_N}\{F_N(q_L l_N) - w_N(q_L, l_N)l_N - k_N v_N - C_N + \frac{\partial \Pi_N(l_N)}{\partial l_N}[p(\theta_N)v_N - \lambda_N l_N]\}, \quad (1)$$

$$\rho\Pi_S(l_S) = \max_{v_S}\{F_S(q^e(Q_d, Q_o)l_S) - w_S(q^e(Q_d, Q_o), l_S)l_S - k_S v_S - C_S + \frac{\partial \Pi_S(l_S)}{\partial l_S}[p(\theta_S)v_S - \lambda_S l_S]\}. \quad (2)$$

Equations (1) and (2) have similar interpretations. Let's focus only on the second one. It tells us that a firm matched with l_S workers of average quality $q^e(Q_d, Q_o) = E[q \mid Q_d \leq q \leq Q_o]$ produces $F_S(q^e(Q_d, Q_o)l_S)$ units of the final consumption good per period. The firm pays $w_S(q^e(Q_d, Q_o), l_S)$, as educated workforce wage rate and C_S, as a fixed cost of production. To open a vacancy, any given company in the skilled sector must spend k_S, as search costs.

The final terms in equation (2) are related to the flow of workers between employment and unemployment status. This flow is defined by: $\dot{l_S} = p(\theta_S)v_S - \lambda_S l_S$, where the first element on the right hand side relates to the rate at which each vacancy becomes occupied. The second term expresses the flow of workers that lose jobs in each period of time.

Assume that $F_N(q_L, l_N) = q_L l_N^{\alpha_N}$ and $F_S(q^e(Q_d, Q_o), l_S) = q^e(Q_d, Q_o) l_S^{\alpha_S}$ represent the production technologies used in the unskilled and skilled sectors, respectively. The set of conditions that characterize the optimal firm decisions are given by:

$$k_N - \frac{\partial \Pi_N(l_N)}{\partial l_N} p(\theta_N) = 0, \tag{3}$$

$$k_S - \frac{\partial \Pi_S(l_S)}{\partial l_S} p(\theta_S) = 0, \tag{4}$$

$$\begin{aligned} \rho \frac{\partial \Pi_N(l_N)}{\partial l_N} &= \alpha_N q_L l_N^{\alpha_N - 1} - w_N(q_L, l_N) - w_N^{'}(q_L, l_N) l_N \\ &\quad - \frac{\partial \Pi_N(l_N)}{\partial l_N} \lambda_N + \frac{\partial^2 \Pi_N(l_N)}{\partial l_N^2} [p(\theta_N) v_N - \lambda_N l_N], \end{aligned} \tag{5}$$

$$\begin{aligned} \rho \frac{\partial \Pi_S(l_S)}{\partial l_S} &= \alpha_S q^e(Q_d, Q_o) l_S^{\alpha_S - 1} - w_S(q^e(Q_d, Q_o), l_S) - w_S^{'}(q^e(Q_d, Q_o), l_S) l_S \\ &\quad - \frac{\partial \Pi_S(l_S)}{\partial l_S} \lambda_S + \frac{\partial^2 \Pi_S(l_S)}{\partial l_S^2} [p(\theta_S) v_S - \lambda_S l_S]. \end{aligned} \tag{6}$$

From first order conditions (3) and (4), it can be shown that:

$$\frac{\partial^2 \Pi_N(l_N)}{\partial l_N^2} = \frac{\partial^2 \Pi_S(l_S)}{\partial l_S^2} = 0. \tag{7}$$

By using expressions (3) and (4), together with equation (7), in the envelope conditions (5) and (6), we arrive at:

$$\frac{k_N(\rho + \lambda_N)}{p(\theta_N)} = \alpha_N q_L l_N^{\alpha_N - 1} - w_N(q_L, l_N) - w_N^{'}(q_L, l_N) l_N, \tag{8}$$

$$\frac{k_S(\rho + \lambda_S)}{p(\theta_S)} = \alpha_S q^e(Q_d, Q_o) l_S^{\alpha_S - 1} - w_S(q^e(Q_d, Q_o), l_S) - w_S^{'}(q^e(Q_d, Q_o), l_S) l_S. \tag{9}$$

These two previous equations determine the equilibrium values of θ_N and θ_S, characterizing the equilibrium labor demand. They are also very similar. Focusing only on the first equation, the left-hand side represents the expected cost of occupying a type N vacancy. The other side of the expression is related to the expected profit associated to the creation of an additional vacancy. The equilibrium value of θ_N is established in order to equate these two expected returns.

It is important to point out that an increase in the wage rate, a reduction in the job creation costs or a fall in $q^e(Q_d, Q_o)$, they all come with a decrease in θ_S.

The usual hypothesis of free entry and exit conditions assures us that in equilibrium, all economic rents from opening vacancies are exhausted. Then,

$$\alpha_N q_L l_N^{\alpha_N} - C_N - w_N(q_L, l_N)l_N = \frac{k_N \lambda_N l_N}{p(\theta_N)}, \tag{10}$$

$$\alpha_S q^e(Q_d, Q_o) l_S^{\alpha_S} - C_S - w_S(q^e(Q_d, Q_o), l_S)l_S = \frac{k_S \lambda_S l_S}{p(\theta_S)}, \tag{11}$$

are related to the equilibrium zero profit condition in both sectors. These two expressions establish the equilibrium values of l_N and l_S and characterize firms' size in each sector. Notice that the left hand sides of these equations are associated to the firm revenues of employing l workers while the right hand sides give us the firm costs.

3.1.2 Workers

Let $W_N(l_N)$ and $U_N(W_S(l_S)$ and $U_S)$ be the present discounted value of the expected gains associated to employment and unemployment statuses for an unskilled (skilled) worker.

An unemployed worker with schooling cost I who has studied in a school of quality q receives b_S units of the consumption good as unemployment benefits per period. At an instantaneous rate $z(\theta_S)$ the educated unemployed worker finds a vacant job, moving to employment status[16]. In this way we have that:

$$\rho W_N(l_N) = w_N(q_L, l_N) - \lambda_N(W_N(l_N) - U_N), \tag{12}$$

$$\rho U_N = b_N + z(\theta_N)(W_N(l_N) - U_N), \tag{13}$$

[16]Workers who do not study receive b_N units of the consumption good as unemployment insurance and they move to employment state at a rate $z(\theta_N)$.

$$\rho W_S(l_S) = w_S(q, l_S) - \lambda_S(W_S(l_S) - U_S), \quad (14)$$

$$\rho U_S = b_S + z(\theta_S)(W_S(l_S) - U_S), \quad (15)$$

determine the value functions of a non-educated and an educated worker, respectively employed and unemployed in the economy. These expressions are standard in search literature. The first equation implies that a non-educated worker employed in a firm with l_N unskilled workers receives $w_N(q_L, l_N)$ flow units of the consumption good as wages. This employed position is destroyed due to an idiosyncratic shock that occurs at rate λ_N.

Expression (13), in turn, tells us that a worker who studied in his youth receives b_N as unemployment benefits. At rate $z(\theta_N)$ this unemployed worker finds a job vacancy, thus moving into employment status.

If a particular match is destroyed, both the worker and the firm have to pay the costs related to the return to the search process. In this way, a productive match generates a surplus that has to be distributed among the two parties. Consider, as usual in job search theory, that this division is determined by the Generalized Nash Bargain Solution between the firm and the worker, where β_i represents workers' bargaining power in sector $i = \{S, N\}$. The wage rates then satisfy:

$$\beta_N \frac{\partial \Pi_N(l_N)}{\partial l_N} = (1 - \beta_N)[W_N(l_N) - U_N], \quad (16)$$

$$\beta_S \frac{\partial \Pi_S(l_S)}{\partial l_S} = (1 - \beta_S)[W_S(l_S) - U_S]. \quad (17)$$

Observe from these two previous expressions that the surpluses generated by the worker and the firm depend on the worker ability. If the schooling option is preferred to an early entry into the labor force, future matching will be of type S. In this case, the wage rate must satisfy equation (17). However, if the employee decides to be non-educated, his wage rate must satisfy expression (16).

Using expressions (5) - (7) and (12) - (17), the wage rates are respectively given by:

$$w_N(q_L, l_N) = \frac{\beta_N \alpha_N q_L}{\beta_N \alpha_N + (1 - \beta_N)} l_N^{(\alpha_N - 1)} + \rho(1 - \beta_N) U_N, \quad (18)$$

$$w_S(q, l_S) = \frac{\beta_S \alpha_S q}{\beta_S \alpha_S + (1 - \beta_S)} l_S^{(\alpha_S - 1)} + \rho(1 - \beta_S) U_S.$$

These two expressions give us the wage rates considering both individuals who studied and those who didn't. The wage rates are a weighted average of two terms: one is related to workers' job match productivity and the other to the workers' outside options. Since job match productivity varies if workers attended school or not and it is also affected by the quality of the school attended, the first term differs between educated and non-educated workers. Therefore, the higher the quality of the attended school, the bigger the job match productivity and the wage rate.

The average wage rate in the skilled sector is given by:

$$w_S(q^e(Q_d, Q_o), l_S) = \frac{\beta_S \alpha_S q^e(Q_d, Q_o)}{\beta_S \alpha_S + (1 - \beta_S)} l_S^{(\alpha_S - 1)} + \rho(1 - \beta_S) U_S. \tag{19}$$

3.2 Schooling Market

The individual decision to study is taken by comparing the benefits and the costs of the investment in education with the returns obtained in the unskilled sector. Let:

$$\int_0^\infty e^{-\rho t} \rho U_N \, dt,$$

represents the present value of the gains related to an early entry into the labor force as a non-schooled worker. However, if someone with schooling cost $I \in [I_L, I_H]$ decides to study, the discounted present valued of such decision would be given by:

$$\int_0^T -e^{-\rho t} \rho I q(I) \, dt + \int_T^\infty e^{-\rho(t-T)} \rho U_S \, dt,$$

where the first term is associated to the schooling costs materialized during an exogenous period of studies T. The following term refers to the benefits of being an educated worker. From the two previous expressions we have that whenever:

$$\int_0^T -e^{-\rho t} I q(I) \, dt + \int_T^\infty e^{-\rho(t-T)} U_S \, dt \geq \int_0^\infty e^{-\rho t} U_N \, dt,$$

the agent I decides to study.

Assume that $Q_d(I) \in (q_L, q_H]$, for all schooling cost[17]. The school quality reser-

[17] This condition excludes the following limit cases: only individuals with the lowest cost of education study, $Q_d(I_L) = q_H$, and all agents decide to study, $Q_d(I_H) = q_L$.

vation value that leaves an individual I indifferent between study and work activities $Q_d(I)$ satisfies:

$$Q_d(I) = \frac{U_S - U_N}{(1 - e^{-\rho T})I}. \tag{20}$$

The previous expression establishes the minimum public school quality compatible with the indifference between studying and working options. It fully characterizes the individual demand of education[18].

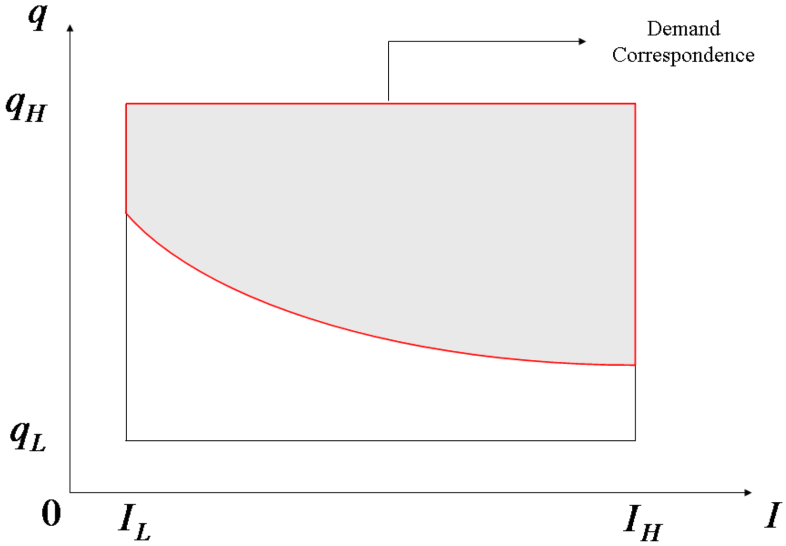

Figure 8: Demand Correspondance

Notice that if the school quality offered by the government is defined in the set $[Q_d(I), q_H]$, individuals will always study, becoming skilled workers after T periods. However, if the school quality received is in the set $[q_L, Q_d(I))$, the individual with cost I will never study. Figure 8 presents the aggregate demand for education.

It is worth noting that the demand of education depends on both labor market as the schooling market variables. An increase in the wage rate on the unskilled sector, for instance, reduces the demand of education and an improvement on the school quality distribution increases the demand of education[19].

[18] Notice that $Q_d(I)$ is continuous differentiable in $[I_L, I_H]$, $Q'_d(I) < 0$ and $Q''_d(I) > 0$.

[19] We will refer to these two outcomes as the push and the pull effect, respectively. We will return to this point later.

Now consider the supply of education. As previous mentioned, the government policy is characterized by the following rule: first, it observes the individual schooling cost, I. Then, the government defines the set of school places available to each agent I, $[q_L, Q_o(I)]$. Consider that this set is defined by:

$$Q_o(I) = q_H - (q_H - q_L)(\frac{I - I_L}{I_H - I_L})^{\epsilon}, \tag{21}$$

where the term ϵ captures the benevolence of the school supply policy.

Notice that the higher is the term ϵ, the bigger is school quality $Q_o(I)$ and the set of school options available to each individual I, $[q_L, Q_o(I)]$. In this way, ϵ can be seen as an school enrollment policy parameter. The bigger it is, the greater is the government expansion of education sector by enlarging the set of school vacancies available to each individual.

It can also be shown from the previous expressions that the individual with the highest schooling cost in the economy, I_H, receives the lowest school quality available, $Q_o(I_H) = q_L$. In turn, the agent with lowest schooling cost, I_L, always receives the best school option available in the economy, $Q_o(I_L) = q_H$. Figure 9 characterizes the aggregate supply for education in the economy. It also presents the equilibrium in the schooling market.

It can be shown that there exist a unique $\tilde{I}$ that characterizes the set of skilled workers in the economy, $Q_o(\tilde{I}) = Q_d(\tilde{I})$. If $I \leq \tilde{I}$, then $Q_d(I) \leq Q_o(I)$ and the agent with cost I will always become schooled. However, if $I > \tilde{I}$, then $Q_d(I) > Q_o(I)$. In this last case, the school offers does not meet the minimum individual quality requirement to study, $Q_d(I)$. So the agent decides to become unskilled.

In sum, an individual that was born with schooling cost I will have ability given by:

$$q(I) \begin{cases} \in [Q_d(I), Q_o(I)] & \text{if } I \leq \tilde{I} \\ = q_L & \text{otherwise.} \end{cases}$$

.

3.3 Aggregation and the Decentralized Equilibrium

Let Q_d and Q_o represent the aggregate school quality demanded by individuals and the aggregate school quality supplied by the government, respectively. We have that:

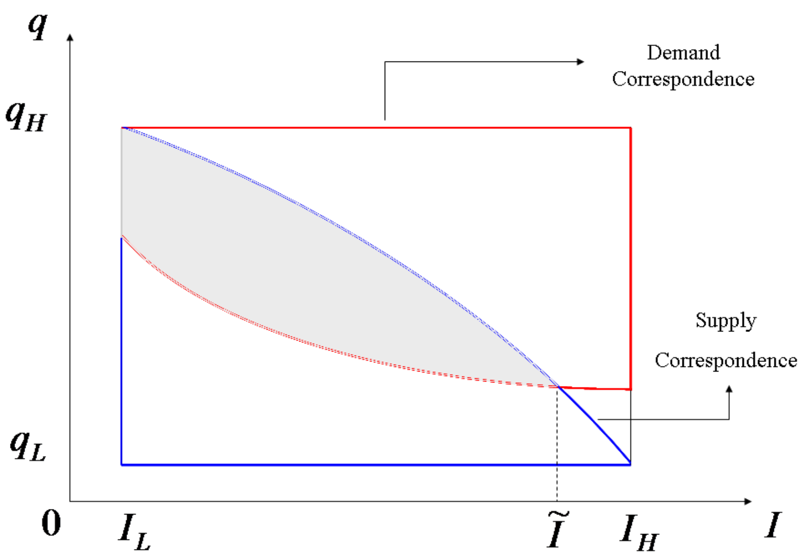

Figure 9: Schooling Market Equilibrium

$$Q_d = E[Q_d(I) \mid I \leq \tilde{I}] = \int_{I_L}^{\tilde{I}} \frac{U_S - U_N}{(1-e^{-\rho T})I} \frac{dH(I)}{H(\tilde{I})}, \tag{22}$$

$$Q_o = E[Q_o(I) \mid I \leq \tilde{I}] = q_H - (q_H - q_L) \int_{I_L}^{\tilde{I}} (\frac{I - I_L}{I_H - I_L})^{\epsilon} \frac{dH(I)}{H(\tilde{I})}, \tag{23}$$

fully characterize the aggregate quality of the schooled labor force in the economy, $q^e(Q_d, Q_o) = E[q \mid Q_d \leq q \leq Q_o]$.

The previous equilibrium expressions (22) and (23) deserve some comments. Let μ represents the mean of the school quality distribution, $G(q)$. It can be shown that:

(i) $\lim_{Q_d \to q_H} q^e(Q_d, Q_o) = q_L$, if $Q_o = q_L$;
(ii) $\lim_{Q_d \to q_L} q^e(Q_d, Q_o) = \mu$ and $\lim_{Q_d \to q_H} q^e(Q_d, Q_o) = q_H$, if $Q_o = q_H$[20].

These two results guarantee that there will be no schooled sector in the economy if the public supply of education is given by $Q_o = q_L$. They also assure, if $Q_o = q_H$, that is as the size of the labor force that decides to study goes to the unit, the average productivity of the skilled sector converges to the mean of the school quality

[20] We only need to apply L'Hopital Rule to prove these results.

distribution, μ. In turn, as the size of the non-educated labor force converges to the unit, the average productivity of the educated workforce moves to the highest value of the distribution, q_H[21].

Another interesting aspect of the aggregate quality of the skilled labor force is:

$$\frac{\partial q^e(Q_d, Q_o)}{\partial Q_d} = \frac{q^e(Q_d, Q_o) - Q_d}{G(Q_o) - G(Q_d)} g(Q_d) > 0;$$

$$\frac{\partial q^e(Q_d, Q_o)}{\partial Q_o} = \frac{Q_o - q^e(Q_d, Q_o)}{G(Q_o) - G(Q_d)} g(Q_o) > 0;$$

since $Q_d < q^e(Q_d, Q_o) < Q_o$. Thus, any policy that reduces the demand of education (i.e. increases Q_d) implies a raise in $q^e(Q_d, Q_o)$. In turn, any policy that reduces the supply of education implies the opposite effect.

Proposition 1 *Consider an increase in the enrollment policy parameter ϵ. It implies:*

(i) A reduction in Q_d;

(ii) An increase in Q_o.

Proof. See Appendix A. ■

The previous proposition guarantee that an increase in the school enrollment - through a higher value of ϵ - implies both an increase in the demand and in the supply of education. The latter impact is direct whilst the former one is indirect and occurs through a higher equilibrium value of $\tilde{I}$.

Notice that the higher is ϵ, the lower is the equilibrium value of Q_d and the greater is the increase in the aggregate demand of education. Q_o also increases with a higher ϵ. In this way, the school enrollment policy may have an ambiguous impact over the average quality of skilled labor force, $q^e(Q_d, Q_o)$. On the one hand, it increases the average quality through a high value of Q_o. On the other hand, it decreases the average quality $q^e(Q_d, Q_o)$, through a smaller value of Q_d.

Notice that the composition effect may not exist in our model. It suffices to consider that the positive impact is greater than the negative indirect one.

[21]Consider that the schooling distribution is non-degenerate such that $\mu < q_H$. This result can be used to explain the stylized fact that countries with low educational levels tend to pay higher wage rates to their educated workforce. See Avalos and Savvides (2006), Birdsall, Ross, and Sabot (1995), Bills and Klenow (2000), Patrinos and Psacharopoulos (2004) and references therein on this topic.

Definition 2 *A steady-state equilibrium for this economy is a thirteen-tuple:* $(\theta_i, v_i, l_i, w_i(\cdot), u_i, Q_d, Q_o, \tilde{I})$ *such that:*

(i) $\rho U_i = b_i + \frac{\beta}{1-\beta} k_i \theta_i$, $\theta_i = \frac{v_i}{u_i}$ *and* $p(\theta_i) v_i = \lambda_i l_i$, *for* $i = \{S, N\}$;

(ii) $Q_o(\tilde{I}) = Q_d(\tilde{I})$;

(iii) *equations* (8), (9), (10), (11), (18), (19), (22) *and* (23) *are satisfied.*

The equilibrium has a block recursive structure. First, given the distributions of $G(q)$ and $H(I)$ the equilibrium value of Q_o is obtained. Then, individuals determine the aggregate demand of education, Q_d, and firms the equilibrium labor demand in each sector. The remaining labor market equilibrium variables follows.

Notice that using expressions (18) and (8) we obtain the equilibrium values of $w_N(\cdot)$ and θ_N. It can be seen that the equation that characterizes the equilibrium value of θ_N does not depend on θ_S and $q^e(Q_d, Q_o)$. By using a similar reasoning, equations (19) and (9) determine the equilibrium value of $w_S^e(\cdot)$ and θ_S. They are both functions of $q^e(Q_d, Q_o)$. The expressions that characterize the equilibrium values of θ_N, θ_S, Q_d and Q_o are respectively given by:

$$\frac{k_N(\rho + \lambda_N + \beta_N z(\theta_N))}{p(\theta_N)} = \left\{ \frac{(1-\beta_N)(1-\alpha_N)^{(1-\alpha_N)}}{(1+\rho)C_N[\beta_N \alpha_N + (1-\beta_N)]} \right\}^{\frac{1}{\alpha_N}} \alpha_N q_L^{\frac{1}{\alpha_N}} - (1-\beta_N) b_N;$$

$$\frac{k_S(\rho + \lambda_S + \beta_S z(\theta_S))}{p(\theta_S)} = \left\{ \frac{(1-\beta_S)(1-\alpha_S)^{(1-\alpha_S)}}{(1+\rho)C_S[\beta_S \alpha_S + (1-\beta_S)]} \right\}^{\frac{1}{\alpha_S}} \alpha_S q^e(Q_d, Q_o)^{\frac{1}{\alpha_S}} - (1-\beta_S) b_S;$$

$$Q_d = \frac{(b_S + \frac{\beta_S}{1-\beta_S} k_s \theta_S) - (b_N + \frac{\beta_N}{1-\beta_N} k_n \theta_N)}{\rho(1-e^{-\rho T})} \frac{dH(I)}{H(\tilde{I})};$$

$$Q_o = q_H - (q_H - q_L) \int_{I_L}^{\tilde{I}} (\frac{I - I_L}{I_H - I_L})^{\epsilon} \frac{dH(I)}{H(\tilde{I})}.$$

4 Quantitative Analysis

In this section, we assess the impact of a school enrollment policy on the decentralized labor market. Namely, we are interested in evaluating the impact of an increase on the educational parameter ϵ on the labor and schooling markets, measuring their effects on the trade-off between school attendance and the early entry into the labor force.

We calibrate the benchmark model to match the main empirical regularities of the Brazilian economy. That is, we provide numerical evaluations of an inclusive

A. Fixed Parameters		
Parameters	**Values**	**Comment**
$\beta_N = \beta_S$	0.5	Bargaining Power
$\lambda_N = \lambda_N$	0.75	Separation Rate
$\alpha_N = \alpha_S$	0.65	Labor Share
$k_N = k_S$	1.40	Job Creation Costs
T	0.1280	9 Years of Schooling
ρ	0.9839	Quarterly Real Interest Rate, 2002-2015
μ	4.24015	Parameter - Schooling Cost Distribution
σ	1.49816	Parameter - Schooling Cost Distribution
γ	0.5	Elasticity - Matching Function
B. Calibrated Parameters		
Parameters	**Values**	**Comment**
c_N	1	Fixed Costs - Normalization
c_S	3.2708	Fixed Costs
b_N	0.80	Unemployed Benefits - Normalization
b_S	0.92	Unemployed Benefits
q_L	1	Productivity - Normalization
q_H	4	Productivity
ϵ	3	School Supply Parameter

Table 4: Parameter Values

public educational policy on: the aggregate demand of education, the average quality of the skilled labor force, the mass and the wage rate of the two type of workers. We also provide a quantitative evaluation of the impact of a higher enrollment parameter ϵ on the ratio of jobs vacancies at the two economic sectors of the previous section.

4.1 Calibration

In order to perform the model simulations, we first need to determine all endogenous and exogenous model parameters and set the functional forms of our benchmark economy.

The time period is a quarter and the discount factor is set to 0.9839 in order to match the 6.73% average real interest rate during the period 2002-2015[22]. Assume, without loss of generality, that all individuals must complete the ninth grade of schooling to become an skilled worker[23].

Each individual lives, on average, for 70 years. This represents a total life of 280 quarters and 12.80% of an entire lifetime being dedicated to human capital

[22]The real interest rate represents the difference between the monthly primary rate established by the Brazilian Central Bank - SELIC rate - and the consumer price index - INPC - from IBGE. The number used in simulations corresponds to the quarterly average over the period.

[23]Notice that according to section 2 there is an increase in the school dropout rate ate the end of each stage of educations. We consider, without loss of generality, the first increase in the dropout rate. Namely, the last year of the primary education.

accumulation to become an skilled worker.

The difference between the two sectors is very subtle once we consider the ninth grade as the year of schooling that split the skilled and unskilled segment of the economy.

The calibrated economy needs also to account for the difference in school enrollment coming from the labor and the schooling markets. In other words, we need to simulate the economy in order to evaluate the labor market pull effect, that attracts unskilled workers to the labor force, and the school quality effect that pushes individuals out of the schools, due to their low average quality, for instance. We also need to evaluate the link between the labor and the schooling markets.

Notice from the theoretical model that the higher the supply of job vacancies and the wage rate for low-quality workers, the greater the school dropout and the early entry into the workforce. In turn, the lower the individual schooling costs or the higher the school quality provision, the lower will be the school dropout and the mass of unskilled workers in the economy.

To account for these previous effects, we fix the job creation costs on 1.40, on both sectors, and set the Brazilian labor share at $\alpha_N = \alpha_S = 0.65$, as proposed by Ulyssea (2010). The job destruction rate is set to 0.75 as suggested by Bosch and Esteban-Pretel (2012). We also assume that the bargained power is given by $\beta_N = \beta_S = 0.5$, which is standard in search literature.

As usual in search literature the matching function is given by a Cobb-Douglas function $m(v_i, u_i) = v_i^{1-\gamma} u_i^{\gamma}$, where γ represents the elasticity of the matching function with respect to unemployment. We follow Charlot, Malherbet, and Terra (2012) and references therein to set the matching elasticity in 0.5.

The schooling cost distribution is lognormal whilst the school quality distribution is assumed to be uniform. The parameters values μ and σ of the lognormal distribution are found by the maximum likelihood estimation of the total educational costs from the Brazilian Household Budget Survey. They are given by 4.24015 and 1.49816, respectively.

To find the remaining model parameters, we make use of the Brazilian datasets presented on section 2. We target the ratio of skilled to unskilled individuals in the Brazilian economy to 5.15. In turn, the dispersion of the school quality distribution is given by 1.65, which represents the ratio of the ninetieth to the tenth percentile of the school quality distribution. Finally, the ratio of the skilled to the total unskilled labor market productivity is target to 1.15.

Given these previous targets, we normalize the fixed production cost, the labor market productivity and the unemployment benefits of the non-skilled sector and

	Model A.	Model B.
Schooling Market		
$Mean(Q_o)$	2.13	1.70
$Std.Dev.(Q_o)$	0.81	0.53
$Mean(Q_d)$	0.87	0.93
$Std.Dev.(Q_d)$	0.57	0.71
P_{10}	1.49	1.41
P_{90}	3.15	2.29
P_{90}/P_{10}	2.11	1.62
Labor Market		
$q^e(Q_o, Q_d)$	1.13	1.05
w_S/w_N	1.06	1.02
$H(\tilde{I})/1 - H(\tilde{I})$	5.40	5.67
u_S/u_N	0.88	0.71
θ_S/θ_N	2.25	2.25
$z(\theta_S)/z(\theta_N)$	1.50	1.50

Table 5: Model Solution

derive the endogenous calibrated parameters for the skilled sector of the economy. They are given by 3.2708, 4 and 0.92, respectively. Finally, by using the fact that the ratio of employed on the skilled sector to the employed on the non-skilled sector is given by 6.0676 in Brazil we calibrate the parameter ϵ. Table 4 summarizes all the parameters values used on the model solution.

4.2 School Enrollment Policy

The benchmark economy is solved on the steady state under the previous parameters values. The main model solutions are presented on the first column of Table 5 (Model A).

The procedure used on the numerical exercise is the following. Firstly, we solve for the educational market equilibrium by considering an economy consisting of $10,000$ different agents with respect to their educational costs. In this stage of the numerical solution we disregard the individual labor market equilibrium variables and only consider the equilibrium values of the demand and the supply of education.

Secondly, by using the equilibrium individual values of Q_o and Q_d, we obtain the aggregate skilled sector productivity, $q^e(Q_o, Q_d)$. The remaining labor market variables follows. Notice that this is the aggregation procedure and the last step on the model solution.

The economy solution fits the observed data very well. It should be mentioned that the average productivity of the skilled sector and the ratio of skilled to non-skilled workers are really close to their target values. They are 1.13 and 5.40 at the model solutions and 1.15 and 5.15 at the observed data, respectively. The school provision distribution also does a relative good job. The average school quality provision stood at 2.13. The dispersion, measured by the ratio of the top 10% schools and 10% worst schools, is simulated on 2.11. The target value for the Brazilian economy is 1.65.

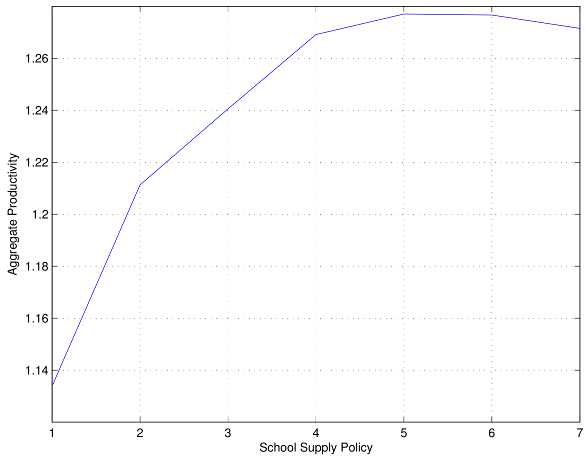

Figure 10: Aggregate Productivity

Figure 10 summarizes the impact of a higher provision of school vacancies on the labor market. It characterizes the impact of an student inclusion policy on the skilled sector productivity.

A larger value of ϵ comes up with a greater supply of school quality, a higher aggregate productivity and a positive impact on the job creation dynamics on the skilled sector. The composition of jobs, measured by the ratio of skilled to unskilled jobs, increases[24]. However, this policy has also an indirect negative effect on the labor market. Since it increases the aggregate demand of education there is also a reduction on the average quality of skilled workforce. This last effect implies a fall on the wage rate and a reduction on the market tightness of the skilled sector.

It can be seen that the expansion on the mass of educated agents may have a strong effects on the economy. The aggregate labor market productivity increases as the mass of individuals that decides to study increases. However, the continuous increase on the mass of students may becomes negative to the labor market. In extreme cases, it may leads to a fall in the skilled sector productivity. In the simulated economy, this happens with a policy parameter value near $\epsilon = 90$.

The composition effect in our model is due to the impact of a higher ϵ on the economy. Since we are evaluating the impact of a school enrollment policy in a

[24]The set of the values of ϵ used on the simulation are $\{3, 6, 9, 30, 60, 90, 300\}$.

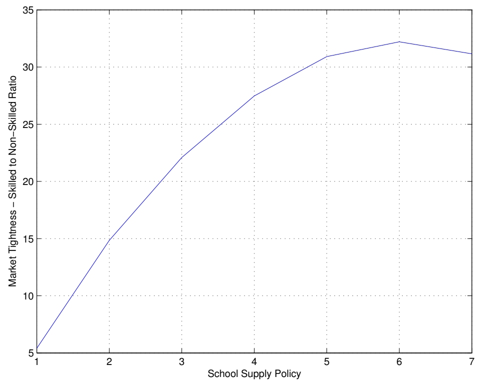

Figure 11: Market Tightness - Ratio

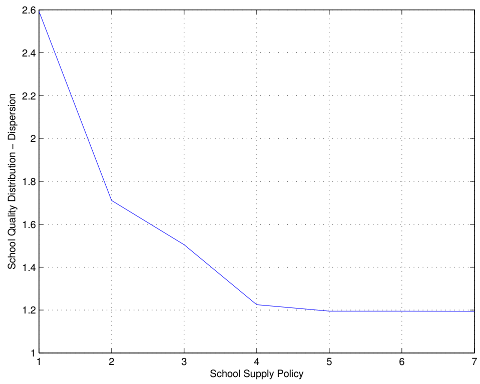

Figure 12: School Quality Distribution

model with a fixed school quality distribution, the bigger the demand of education, the lower the average productivity of the skilled labor force. Then, although the schooling rate might increases with a higher value of ϵ, the aggregate productivity reduces, decreasing the job creation on the high-quality sector and the aggregate composition of jobs in the economy. This previous impact can be confirmed by Figure 11. The ratio of θ_S/θ_N increases, with a higher value of ϵ. However, as the demand for education increases further, the average productivity in the skilled sector begins to fall, leading to a drop in θ_S/θ_N.

Figure 12 guarantees that as the set of school qualities available to each agent increases, there is a reduction on the unequal provision of school vacancies. Notice that although the public education policy has become more egalitarian, with higher values of ϵ, the counterpart is the strong negative effect on the skilled labor market.

4.3 Sensitivity Analysis

The previous section shows the impact of the school enrollment expansion policy on the labor market. In this section, we explore the consequences of using a lognormal schooling cost distribution on the main model results. The main motivation of the current exercise is to answer question: the previous results depends on the the school quality distribution used on the numerical exercise? In other words, we are interested in evaluate the sensitivity of our previous to a different specification of $H(I)$.

Consider, without loss of generality, that the schooling costs follow a Type XII Burr distribution. Then, a new fit of the Brazilian schooling costs to this new distribution generates the following set of parameters: $\alpha = 59.9631$, $c = 1.17813$ and $k = 0.91501$.

As it can be seen by the second column of Table 5, the recalibrated economy continues to fit the observed data very well. There is now a small reduction on the conditional average productivity of the skilled sector and an increase on the ratio of skilled to non-skilled workers in the economy. It can also be seen a reduction on the ratio of the top 10% schools and 10% worst schools in the economy. The remaining model solutions continue to match the main targets.

5 Centralized Equilibrium and Inefficiency

It is a well know fact that imperfections in the labor market generate inefficient labor market outcomes. The link that exists among labor market tightness and worker and firm transition probabilities implies that bargained wages do not fully internalize the search externality, unless the Hosios Condition is present[25]. This happens basically because once matched, firms and workers do not consider the effects of their decisions on the agents still searching for a productive partner. The consequence is that the equilibrium outcome is socially inefficient.

The present section has two main goals. The first one is to characterize the efficient allocation. Second, we are also interested in evaluating if inefficiency prevail in the decentralized equilibrium.

Assume that the problem of the central planner consist in define the mass of individuals that studies and the volume of jobs to be opened in each sector excluding the aggregate schooling costs. That is, the social planner solves the following problem:

$$\max_{Q,v_S,v_N,l_S,l_N} \mathcal{W}(Q,v_S,v_N,l_S,l_N) = \int_0^\infty e^{-\rho t}\rho[q_L l_N^{\alpha_N} + u_N b_N - k_N v_N - C_N]dt$$
$$+\int_T^\infty e^{-\rho(t-T)}\rho[q^e(Q)l_S^{\alpha_S} + u_S b_S - k_S v_S - C_S]dt - [1-G(Q)]\int_0^T\int_{I_L}^{I_H} e^{-\rho t}\rho I Q(I)dH(I)dt; \tag{24}$$

subject to:

$$p(\theta_i)v_i = \lambda_i l_i \quad \text{and} \quad \theta_i = \tfrac{v_i}{u_i}, \quad \text{for } i=\{S,N\};$$
$$u_N = G(Q) - l_N \quad \text{and} \quad u_S = 1 - G(Q) - l_S.$$

The previous expressions shows that the social planner chooses, in the steady state, the optimal number of job vacancies in both sectors in order to maximize pro-

[25]This condition states that if the firms' bargaining power equalizes the elasticity of the matching function, the decentralized equilibrium is efficient with regards to vacancies. See Pissarides (2000) and Hosios (1990) for more on this subject.

duction subjected to searching costs. The planner also selects the mass of individuals that must be educated, Q.

The first term on the right-hand side of (24) corresponds to the output and the benefits enjoyed by employed and unemployed workers in the unskilled sector. This amount is deduced by the cost of opening a new vacancy in this sector. The following term is identical to the first one, however it refers to the skilled sector. The final term is related to the schooling costs.

The social planner problem can be restated as:

$$\max_{Q,\theta_S,\theta_N} \mathcal{W}(Q, v_S, v_N, l_S, l_N) = G(Q)\left\{q_L[\frac{z(\theta_N)G(Q)}{\lambda_N + z(\theta_N)}]^{\alpha_N}\frac{1}{G(Q)} + \frac{\lambda_N b_N - k_N\lambda_N\theta_N}{\lambda_N + z(\theta_N)} - \frac{C_N}{G(Q)}\right\} +$$

$$[1 - G(Q)]\left\{q^e(Q)[\frac{z(\theta_S)[1 - G(Q)]}{\lambda_S + z(\theta_S)}]^{\alpha_S}\frac{1}{[1 - G(Q)]} + \frac{\lambda_S b_S - k_S\lambda_S\theta_S}{\lambda_S + z(\theta_S)} - \frac{C_S}{[1 - G(Q)]}\right\} -$$

$$[1 - G(Q)]\frac{1}{\rho}[(b_S + \frac{\beta_S}{1 - \beta_S}k_S\theta_S) - (b_N + \frac{\beta_N}{1 - \beta_N}k_N\theta_N)].$$

Consider that the Hosios Condition is fulfilled. Then, the set of expressions that characterizes the social optimum are given by:

$$\frac{k_N(\lambda_N + \beta_N z(\theta_N^P))}{p(\theta_N^P))} = \alpha_N q_L(1 - \beta_N)[\frac{z(\theta_N^P)G(Q^P)}{\lambda_N + z(\theta_N^P)}]^{(\alpha_N - 1)} - \frac{1}{\rho}[\frac{1 - G(Q^P)}{G(Q^P)}]\frac{\beta_N k_N}{(1 - \beta_N)p(\theta_N^P)} - (1 - \beta_N)b_N; \quad (25)$$

$$\frac{k_S(\lambda_S + \beta_S z(\theta_S^P))}{p(\theta_S^P))} = \alpha_S q^e(Q^P)(1 - \beta_S)[\frac{z(\theta_S^P)(1 - G(Q^P))}{\lambda_S + z(\theta_S^P)}]^{(\alpha_S - 1)} + \frac{1}{\rho}[\frac{1 - G(Q^P)}{G(Q^P)}]\frac{\beta_S k_S}{(1 - \beta_S)p(\theta_S^P)} - (1 - \beta_S)b_S; \quad (26)$$

$$\mathcal{A}(\theta_N^P)G(Q^P)^{\alpha_N - 1}q_L + [1 - G(Q^P)]^{\alpha_S - 1}[\mathcal{B}(\theta_S^P)q^e(Q^P) - \mathcal{C}(\theta_S^P)Q^P] = \mathcal{D}(\theta_N^P) - \mathcal{E}(\theta_S^P) - \mathcal{F}(\theta_N^P, \theta_S^P); \quad (27)$$

where:

$$\mathcal{A}(\theta_N^P) = \alpha_N[\frac{z(\theta_N^P)}{\lambda_N + z(\theta_N^P)}]^{\alpha_N}; \quad \mathcal{B}(\theta_S^P) = (1 - \alpha_S)[\frac{z(\theta_S^P)}{\lambda_S + z(\theta_S^P))}]^{\alpha_S};$$

$$\mathcal{C}(\theta_S^P) = \alpha_S[\frac{z(\theta_S^P)}{\lambda_S + z(\theta_S^P))}]^{\alpha_S}; \quad \mathcal{D}(\theta_N^P) = \frac{k_N\lambda_N\theta_N^P - \lambda_N b_N}{\lambda_N + z(\theta_N^P)};$$

$$\mathcal{E}(\theta_S^P) = \frac{k_S\lambda_S\theta_S^P - \lambda_S b_S}{\lambda_S + z(\theta_S^P)};$$

$$\mathcal{F}(\theta_N^P, \theta_S^P) = \frac{1}{\rho}[(b_S + \frac{\beta_S}{1-\beta_S}k_S\theta_S^P) - (b_N + \frac{\beta_N}{1-\beta_N}k_N\theta_N^P)];$$

and θ_S^P, θ_N^P and Q^P represent the efficient values of the market tightness in the skilled and unskilled sectors and the efficient mass of skilled individuals, respectively.

Notice from (25) that the cost of opening a new vacancy in the unskilled sector equalizes the social surplus generated by this sector. This expression defines the optimal steady-state value of θ_N^P. The following equation characterizes the efficient value of θ_S^P. Finally, expression (27) determines the optimal mass of the educated workforce that generates the efficient outcome.

Proposition 3 *Let* $(\theta_N, \theta_S, Q_d, Q_o)$ *represent the equilibrium allocations in the decentralized economy. Consider that:* $\alpha_N = \alpha_S = 1$; $\lambda_N = \lambda_S = 0$; $(1+\rho)C_N = (1+\rho)C_S = 1$; $m(u,v) = u^{\frac{1}{2}}v^{\frac{1}{2}}$; *and the Hosios Condition is satisfied. Then:*

$$\begin{array}{lll} \theta_N \geq \theta_N^P & \text{and} & \theta_S < \theta_S^P; \ \text{if} \ Q_d < Q^P < G^{-1}(\frac{\beta_N}{\rho^2}); \\ \theta_N < \theta_N^P & \text{and} & \theta_S \lesseqgtr \theta_S^P; \ \text{if} \ Q_d > Q^P \geq G^{-1}(\frac{\beta_N}{\rho^2}). \end{array}$$

Proof. See Appendix B. ■

The previous proposition presents a parallel between the social planner and the decentralized equilibrium allocations. It shows, in a scenario with constant returns to scale in the two production functions, the absence of job destruction and the fulfillment of the Hosios Condition, that there are an excessive job creation in the unskilled sector and a reduced job creation in the skilled sector, whenever $Q_d < Q^P$. That is, the number of new jobs opened in the unskilled sector at the decentralized equilibrium is always lower than the efficient number of job vacancies opened in this sector. This happens whenever the decentralized number of educated workers is lower than the efficient number. In turn, if $Q_d > Q^P$ there is always a lack of job creation in the unskilled sector at the decentralized equilibrium. However, the impact of an excess number of educated workers in the skilled sector is ambiguous.

Figure 13 presents an example of this ambiguity over the skilled sector labor market. In the top figure, we present the scenario where the average productivity at the decentralized economy is higher than the efficient productivity, $q^e(Q_d, Q_s) > q^e(Q^P, q_H)$, whilst at the other figure we have that $q^e(Q_d, Q_s) > q^e(Q^P, q_H)$. Notice that although this is not the only element that commands the difference between θ_S and θ_S^P, it highlights the ambiguity that exists between the mass of people that becomes educated and the average quality of the skilled labor force.

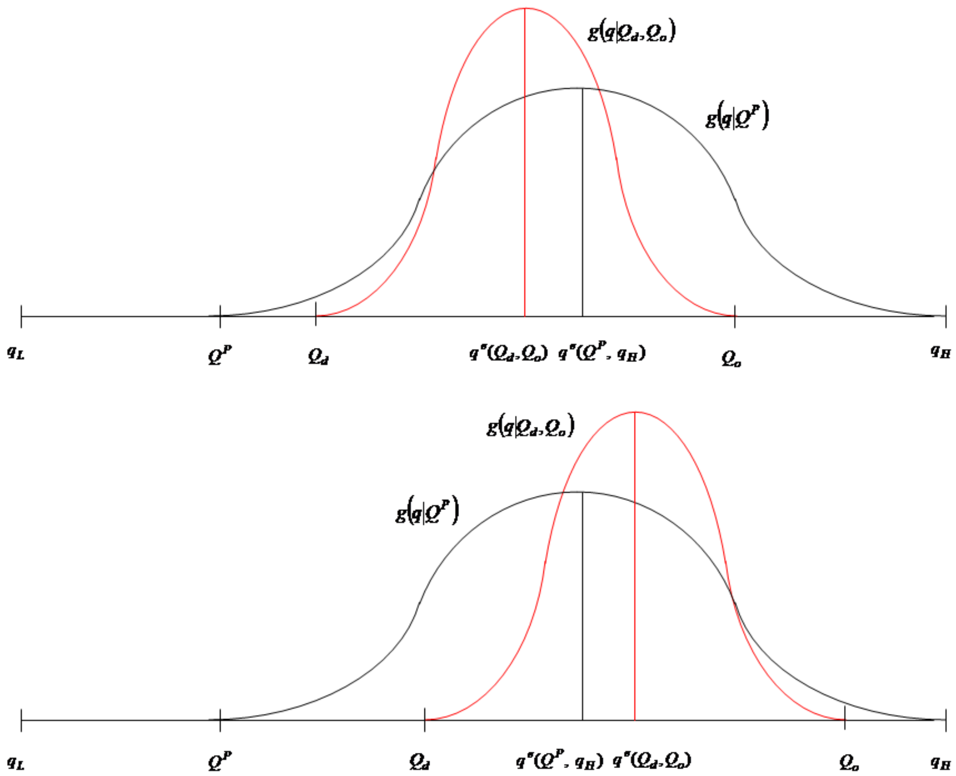

Figure 13: Average Productivity, $Q_d > Q^P$

6 Concluding remarks

The policy that promotes the skilled sector has become widespread in the developing economies. Based on the evidence that government may affect the size of the educated labor force, many countries have spent significant amounts of cash to boost school enrollment and reduce the school drop out rate.

The main objective of this paper is to study the effects of an educational policy reform that aims to increase the aggregate mass of students over the economy. Namely, we are interested in evaluate the impact of an school enrollment policy on productivity and the size of the skilled sector.

We show on a general equilibrium model with labor market frictions that this policy may increase labor market inefficiency and reduce aggregate productivity.

The main mechanism behind this result is the composition effect that may or may not be active.

The more sensitive is the demand of education to the schooling enrollment policy the more evident is the composition effect and the higher is negative effect of the enrollment policy on the economy. Notice that a more inclusive educational policy increase the mass and the quality of skilled labor force. However, there is also an indirect negative effect over these variables, coming from a higher labor demand.

We also calibrate our benchmark economy and provide numerical evaluations for the Brazil. We found evidences that although it is not bad to have a large schooled sector, a policy design to promotes education may not be recommended in some situations. Some individuals that would not study may decide to become educated. This reduces the average productivity of the skilled sector and shrinks the relative size of this segment in the economy.

Appendix

Appendix A

Consider, without loss of generality, that $\epsilon > 1$ in order to guarantee that $Q_o(I)$ is strictly concave. For each ϵ, let $\tilde{I}(\epsilon)$ be the solution of $Q_o(\tilde{I}(\epsilon)) = Q_d(\tilde{I}(\epsilon))$. Now, consider an increase from ϵ to ϵ' and let:

$$Q_o'(I) = q_H - (q_H - q_L)(\frac{I - I_L}{I_H - I_L})^{\epsilon'}.$$

It follows that $Q_o'(I) > Q_o(I)$, for all $I \in (I_L, I_H)$. In particular, we have that

$$Q_o'(\tilde{I}(\epsilon)) > Q_d(\tilde{I}(\epsilon)).$$

However, from the definition of $\tilde{I}(\epsilon')$, we have that:

$$Q_o'(\tilde{I}(\epsilon')) = Q_d(\tilde{I}(\epsilon')).$$

Adding up the last two expressions and multiplying by 1/2, we have:

$$Q_o'(\frac{\tilde{I}(\epsilon) + \tilde{I}(\epsilon')}{2}) > Q_d(\frac{\tilde{I}(\epsilon) + \tilde{I}(\epsilon')}{2}).$$

The inequality follows from the strict concavity of $Q_o'(I)$ and the strict convexity of $Q_d(I)$. From these two last expressions, we have that:

$$\frac{\tilde{I}(\epsilon) + \tilde{I}(\epsilon')}{2} < \tilde{I}(\epsilon')$$

then, $\tilde{I}(\epsilon) < \tilde{I}(\epsilon')$.

Now, let's show that Q_d is decreasing in $\tilde{I}$. Consider that

$$Q_d = M\delta_d,$$

for:

$$M = \frac{\frac{1}{\rho}[(b_S + \frac{\beta_S}{1-\beta_S}k_S\theta_S) - (b_N + \frac{\beta_N}{1-\beta_N}k_N\theta_N)]}{(1-e^{-\rho T})};$$

$$\delta_d = \mathbb{E}[\frac{1}{I}|I \leq \tilde{I}].$$

Notice that changes in Q_d, due to changes in $\tilde{I}$, occurs through the term δ_d. We may rewrite δ_d as:

$$\delta_d = \mathbb{E}[x|x \geq \tilde{x}].$$

Then, increases in $\tilde{I}$ are equivalent to a reduction on $\tilde{x}$ and on δ_d. This guarantees that the higher is ϵ, the lower is Q_d.

Now let's verify the impact of ϵ on Q_o. Consider that:

$$Q_o = q_H - (q_H - q_L)\delta_o$$

for

$$\delta_o = \mathbb{E}[(\frac{I - I_L}{I_H - I_L})^\epsilon|I \leq \tilde{I}].$$

As previously, consider that:

$$x = \frac{I - I_L}{I_H - I_L} \in [0, 1]$$

for all $I \in [I_L, I_H]$. Then,

$$\delta_o = \int_0^{\tilde{x}} x^\epsilon \frac{d\tilde{H}(x)}{\tilde{H}(\tilde{x})},$$

where $\tilde{H}(x)$ is the distribution of transformation $x = (I - I_L)/(I_H - I_L)$.

Differentiating this last expression with respect to ϵ, we get

$$\frac{d\delta_o}{d\epsilon} = \left[(\tilde{x}^\epsilon - \delta_o)\tilde{h}(\tilde{x}) + \int_0^{\tilde{x}} x^\epsilon ln(x)\frac{d\tilde{H}(x)}{\tilde{H}(\tilde{x})}\right]\frac{1}{\tilde{H}(\tilde{x})}\frac{d\tilde{x}}{d\epsilon} < 0.$$

The sign of the previous expression is derived from: $\tilde{x}^\epsilon - \delta_o < 0$, $0 < x < 1$ and

$d\tilde{x}/d\epsilon \propto d\tilde{I}/d\epsilon$.

Appendix B

(i) Let

$$\frac{(\rho+\beta_N q(\theta_N))k_N}{p(\theta_N)} + (1-\beta_N)b_N = \frac{(1-\beta_N)q_L}{(1+\rho)C_N},$$

$$\frac{\beta_N z(\theta_N^P)k_N}{p(\theta_N^P)} + (1-\beta_N)b_N = (1-\beta_N)q_L - [\frac{1-G(Q^P)}{\rho G(Q^P)}][\frac{\beta_N k_N}{(1-\beta_N)p(\theta_N^P)}].$$

Subtracting one from the other, we have:

$$(A) \qquad (\theta_N^P)^{1/2}\Big\{x - [\frac{1-G(Q^P)}{\rho^2 G(Q^P)}][\frac{\beta_N}{(1-\beta_N)}]\Big\} + (x-1)(x+1)\frac{\beta_N \theta_N^P}{\rho},$$

after considering the Hosios Condition, $\alpha_N = \alpha_S = 1$ and $x = (\theta_N/\theta_N^P)^{1/2}$.
If $G(Q^P) < \beta_N/\rho^2 < \beta_N$, then $1-G(Q^P) > 1-\beta_N$.
In this scenario,

$$\frac{1-G(Q^P)}{1-\beta_N} > \frac{G(Q^P)}{\beta_N} > \rho^2\frac{G(Q^P)}{\beta_N}$$

implies that:

$$\frac{1-G(Q^P)}{\rho^2 G(Q^P)}\frac{\beta_N}{(1-\beta_N)} > 1.$$

So, we have that $x \geq 1$. Otherwise, we would have:

$$x < 1 < \frac{1-G(Q^P)}{\rho^2 G(Q^P)}\frac{\beta_N}{(1-\beta_N)},$$

which contradicts (A). In this way, $\theta_N \geq \theta_N^P$.
Consider ρ/β_S too small. Now, let

$$\frac{(\rho+\beta_S q(\theta_S))k_S}{p(\theta_S)} + (1-\beta_S)b_S = \frac{(1-\beta_S)q^e(Q_o,Q_d)}{(1+\rho)C_S},$$

$$\frac{\beta_S z(\theta_S^P)k_S}{p(\theta_S^P)} + (1-\beta_S)b_S = (1-\beta_S)q^e(q_H,Q^P) + [\frac{1-G(Q^P)}{\rho G(Q^P)}][\frac{\beta_S k_S}{(1-\beta_S)p(\theta_S^P)}].$$

Subtracting one from the other, we have:

$$\frac{\beta_S}{\rho}(\theta_S - \theta_S^P) = (\frac{1-\beta_S}{\rho k_S})[q^e(Q_o, Q_d) - q^e(q_H, Q^P)]$$

$$-[\frac{1}{p(\theta_S)} + \frac{1-G(Q^P)}{\rho^2 G(Q^P)}\frac{\beta_S}{1-\beta_S}\frac{1}{p(\theta_S^P)}] < (\frac{1-\beta_S}{\rho k_S})[q^e(Q_0, Q^P) - q^e(q_H, Q_d)].$$

Therefore, if $Q_d < Q^P$ then $q^e(Q_o, Q_d) < q^e(q_H, Q^P)$ for every $Q_o \in [q_L, q_H]$. According to the above expression this ensures the desired result.

(ii) Subtracting now the second expression by the first one, we have to

$$(B) \qquad \frac{\beta_S}{\rho}(\theta_S - \theta_S^P) = (\frac{1-\beta_S}{\rho k_S})[q^e(Q_o, Q_d) - q^e(q_H, Q^P)]$$

$$-[(\theta_S)^{1/2} + \frac{1-G(Q^P)}{\rho^2 G(Q^P)}\frac{\beta_S}{1-\beta_S}(\theta_S^P)^{1/2}].$$

By using the same reasoning we have that $\theta_N < \theta_N^P$ and

$$(C) \qquad \frac{1-G(Q^P)}{\rho^2 G(Q^P)}\frac{\beta_S}{1-\beta_S} < 1$$

.

Using (C) in (B) we arrive at:

$$\frac{\beta_S}{\rho}(\theta_S - \theta_S^P) + [(\theta_S)^{1/2} + (\theta_S^P)^{1/2}] > (\frac{1-\beta_S}{\rho k_S})[q^e(Q_o, Q_d) - q^e(q_H, Q^P)].$$

Rearranging this expression, we have:

$$\frac{\beta_S}{\rho}[(\theta_S)^{1/2} - (\theta_S^P)^{1/2} + \frac{\rho}{\beta_S}][(\theta_S)^{1/2} + (\theta_S^P)^{1/2}] > (\frac{1-\beta_S}{\rho k_S})[q^e(Q_o, Q_d) - q^e(q_H, Q^P)].$$

Therefore, $q^e(Q_o, Q_d) > q^e(q_H, Q^P)$ and we get the desired result.

References

Acemoglu, D. (1996): "A Microfoundation for Social Increasing Returns in Human Capital Accumulation," *The Quarterly Journal of Economics*, 111(3), 779–804.

Avalos, A., and A. Savvides (2006): "The Manufacturing Wage Inequality in

Latin America and East Asia: Openness, Technology Transfer, and Labor Supply,," *Review of Development Economics*, 10(4), 553–576.

Basu, K., and P. H. Van (1998): "The Economics of Child Labor," *American Economic Review*, 88(3), 412–427.

Becker, G. (1962): "Investment in Human Capital: A Theoretical Analysis,," *Journal of Political Economy*, 70(5), 9–49.

——— (1993): *Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education*. University of Chicago Press.

Bills, M., and P. Klenow (2000): "Does Schooling Cause Growth?,," *American Economic Review*, 90(5), 1160–1183.

Birdsall, N., D. Ross, and R. Sabot (1995): "Inequality and Growth Reconsidered: Lessons from East Asia,," *The World Bank Economic Review*, 9(3), 477–508.

Bosch, M., and J. Esteban-Pretel (2012): "Job Creation and Job Destruction in the Presence of Informal Markets," *Journal of Development Economics*, 98(2), 270–286.

Burdett, K., and E. Smith (2002): "The Low Skill Trap,," *European Economic Review*, 46(8), 1439–1451.

Cahuc, P., and E. Wasmer (2001): "Does Intrafirm Bargaining Matter in the Large Firm?s Matching Model?,," *Macroeconomic Dynamics*, 5(5), 742–747.

Card, D. (1997): *The Causal Effect of Education on Earnings*. In Handbook of Labor Economics, Edited by Orley Ashenfelter and David Card, Vol. III, Chapter 30, Elsevier.

Charlot, O., and B. Decreuse (2005a): "Self-selection in education with matching frictions," *Labour Economics*, 12, 251–267.

Charlot, O., and B. Decreuse (2005b): "Self Selection in Education with Matching Frictions,," *Labour Economics*, 12(2), 251–267.

Charlot, O., and B. Decreuse (2010): "Over-Education for the Rich, Under-Education for the Poor: A Search Theoretic Microfoundation," *Labour Economics*, 17(6), 886–896.

Charlot, O., F. Malherbet, and C. Terra (2012): "Informality in Developing Economies: Regulation and Fiscal Policies," *Journal of Economic Dynamics Control*, 51(1), 1–27.

Dottori, D., F. Estevan, and I. Shen (2013): "Reshaping the schooling system: The role of Immigration,," *Journal of Economic Theory*, 148(5), 2124–2149.

Duflo, E. (2001): "Schooling and Labor Market Consequences of School Construction in Indonesia: Evidence from an Unusual Policy Experiment," *American Economic Review*, 91(4), 795–813.

Eckstein, Z., and K. Wolpin (1999): "Why Youths Drop out of High School: The Impact of Preferences, Opportunities and Abilities," *Econometrica*, 67(6), 1295–1339.

Evans, W. N., and R. M. Schwab (1995): "Finishing High School and Starting College: Do Catholic Schools Make a Difference?," *The Quarterly Journal of Economics*, 110(4), 94–974.

Fernandez, R., and R. Rogerson (1996): "Income Distribution, Communities and the Quality of Public Education,," *The Quarterly Journal of Economics*, 111(1), 135–164.

Foster, A., and M. Rosenzweig (2006): "Technical Change and Human Capital Returns and Investments: Evidence from the Green Revolution," *American Economic Review*, 86(4), 931–953.

Hosios, A. J. (1990): "On the Efficiency of Matching and Related Models of Search and Unemployment,," *The Review of Economic Studies*, 57(2), 279–298.

Munshi, K., and M. Rosenzweig (2006): "Traditional Institutions Meet the Modern World: Caste, Gender, and Schooling Choice in a Globalizing Economy," *American Economic Review*, 96(4), 1225–1252.

OECD (2015): *Education at a Glance, 2015. Brazil country report.* OECD Publishing.

Patrinos, A., and G. Psacharopoulos (2004): "Returns to Investment in Education: A Further Update,," *Education Economics*, 12(2), 111–134.

Pissarides, C. (2000): *Equilibrium Unemployment Theory*. MIT Press, USA.

Smith, E. (1999): “Search, Concave Production, and Optimal Firm Size,,” *Review of Economic Dynamics*, 2(2), 456–471.

Ulyssea, G. (2010): “Regulation of Entry, Labor Market Institutions and the Informal Sector,” *Journal of Development Economics*, 91(1), 87–99.

UNESCO (2011): *The State of Education in Latin America and Caribbean: Guarateeing Quality Education for All.* Unesco, Santiago, Chile.