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Strong or Patient? An Evaluation of the Bidders in a First-Price Asymmetric Auction with Resale

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STRONG OR PATIENT? AN EVALUATION OF THE BIDDERS IN A FIRST-PRICE ASYMMETRIC AUCTION WITH RESALE

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ABSTRACT

We extend the model of first price auction with resale of Hafalir and Krishna (2008) by allowing that resale is take place through a two-stage model of bargaining. With this setup we find that be strong at the auction stage and be patient at the resale stage are characteristics directly related.

Keywords: asymmetric auctions; resale; bargaining.

JEL Classification: C78; D44; D82

1. INTRODUÇÃO

Asymmetric auctions constitute an important class of models linking theory and real-world practice[1]. In turn asymmetries could be associated to several exogenous factors like differences in budget or opportunity cost of acquire and staying with the auctioned object. Whenever these factors are common knowledge, beliefs will be asymmetric among the bidders.

There are at least two immediate consequences when asymmetries are presents in auctions. First, the Revenue Equivalence Theorem is no longer valid, since that, symmetry is required as assumption. Second, a closed-form solution to equilibrium bid is possible for very particular cases only. This is the case for sealed-bid first price auction.

The lack of a closed-form solution in asymmetric first price auction difficult the characterization of bidding behavior in equilibrium. However, many authors like Lebrun (1998, 1999) and Maskin and Riley (2000), uses the differential equations system from the first order conditions to characterize the equilibrium.

In many applications this technique is employed but often the authors hold some assumptions. The most used assumption is stochastic dominance on bidders beliefs. Specifically is supposed that the distributions of the bidders values obeys a stochastic order. Other assumption commonly held is on the number of participants, often limited to just two. In this case, the assumption of stochastic dominance assumes a more intuitive form.

In a terminology introduced by Maskin and Riley (2000) setting an auction with two bidders, the bidder with value stochastically higher is called the "strong bidder" in opposite to other bidder called the "weak bidder". This order ex ante on the distributions of bidders value makes possible to arrive at several results about bidding behavior in equilibrium. For example, can be showed that in a first price auction the weak bidder bids more aggressively than the strong bidder[2], and the strong bidder strictly prefers the open auction while the weak bidder prefers first-price auction[3].

In short, in absence of a closed-form solution for equilibrium bid, a model with two buyers in which is supposed a stochastic order on beliefs, a lot of interesting results can still be reached. However qualify bidders as strong or weak remains an assumption a priori. In this notes we found a way to make this qualification endogenous.

The model used in this paper is a version of the first-price auction with resale proposed by Hafalir and Krishna (2008) which in turn has a similar structure to the model of Gupta and Lebrun (1999). Both models share the same mechanism of resale that consists in a Take-it-or-Leave-it offer and the future is entirely discounted. In Gupta and Lebrun (1999) the

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[1] See Flambard and Perrigne (2006) for an example.
[2] See the Proposition 4.4 in Krishna (2002).
[3] See the Proposition 3.6 in Maskin and Riley (2000).

authors assume that, regardless of outcome, all information about the true values of bidders is publicly announced. On other hand, in Hafalir and Krishna (2008) it is supposed that the only information revealed at the end of auction is the winner bid. Despite of difference on device of information disclosure, Gupta and Lebrun (1999) and Hafalir and Krishna (2000) reach an outstanding result: resale symmetrize the auction.

We extend the model by setting that resale is take place through a two-stage repeating bargaining. Furthermore bidders discount the future at the different rates. These modifications introduce additional complications in the model because the simmetrization result becomes a particular case. But it is exactly this generalization that allows us to reach at the main result of this paper. As shown in the Theorem 3, the more patient participant at the resale stage will be the strong bidder at the auction stage. Thus whenever that the researcher find necessary qualify the bidders as strong or weak, he can, if the model allow, justify your choice by identifying the strong bidder as the more patient participant in a possible trade post-auction.

2. THE MODEL

We consider a first-price auction with two risk-neutral agents. After the auction, is supposed that there is a secondary market where the auction's winner can resale the object to other bidder. We assumes that the bidders has private and independent values, but asymmetrically distributed, such that

$$X_i \sim F_i[0, \omega_i]$$

where X_i is a random variable representing the maximum willingness-to-pay of bidder *i* for the object; F_i is the absolutely continuous distribution function on X_i and $[0, \omega_i]$ is the support of

F_i , so that $F_i^{'} = f_i > 0$ on $[0, \omega_i]$.

After the auction and before the resale stage only the winner bid is announced. Just like argumented in Hafalir and Krishna (2008), this is standard in the major of sealed bid auctions. In the resale stage the winner and the looser of auction can (if want) negotiate the object. The negotiation is take place through a model of finite repeating bargaining just like found at Sobel and Takahashi (1983). Specifically we have that:

1. The seller has all bargaining power and makes all offers;
2. The seller don't observe the value of the buyer but updates your belief at the auction result;

3. Seller and buyer has discount rates given by δ_i and δ_j such that $\delta_\lambda \in (0,1]$ for $\lambda = i, j$;
4. No additional trade is possible at the end of bargaining, doesn't matter which result was reached.

The solution concept for the game form is the Perfect Bayesian Equilibrium, ie: strategies are required to yield a Bayesian equilibrium in every stage given posterior beliefs of the players, and beliefs are required to be updated in accordance with Bayes' law whenever it is applicable. We work with a simplified version where the bargaining game has just two periods and the seller sets the price of the object under the commitment of follow the announced strategy.

2.1 Resale

Let a bidder i and suppose that he uses an equilibrium strategy at the auction that is a continuous and increasing function denoted by β_i such that $\beta_i^{-1} = \phi_i$. We admit without proof that $\beta_i(0) = \beta_j(0) = 0$ and $\beta_i(\omega_i) = \beta_j(\omega_j) = \overline{b} > 0$. Now, suppose that the bidder i win the auction with a bid b. The winner doesn't observe the loser's bid but infers that $\beta_j(X_j) \leq b$ implying that $X_j \leq \phi_j(b)$. Thus, if $x_i < \phi_j(b)$ and i win the auction with b, mutual gains rising at the resale stage with positive probability. Let $G(x)$ be the i´s posterior beliefs that j's value is less or equal to x, so that

$$G(x) = \frac{F_j(x)}{F_j(\phi_j(b))} \text{ for any } x \in [0, \phi_j(b)]$$

In the sequel we define the commitment equilibrium of first price auction with resale.

Definition 1 *A commitment equilibrium in the asymmetric first-price auction with resale is a menu of prices* $(b, z_1(x), z_2(x))$ *and belief* G *that maximizes the expected payoff of the participants. In the equilibrium menu,* $b = \beta(x)$ *is the winner bid at the auction stage and* x *is the value of auction winner;* $z_t(x)$ *is the price charged by the auction winner for the period* $t = 1,2$ *at the resale stage*

In the bargaining stage the seller makes all offers once he/she act as a monopolist. However, the seller is commit himself to a sequence of optimal prices doesn't matter the actions of his opponent. In this way, the commitment equilibrium sequence of prices is obtained maximizing the discounted expected payoff of seller.

Fix the bidder *i* and suppose he win the auction bidding $b = \beta_i(x_i)$ such that $x_i < \phi_j(b)$ (henceforward, we hold this assumption). Then at the beginning of bargaining stage the discounted expected revenue of seller is given by

$$\begin{aligned} & z_1 \Pr\left[X_j - z_1 > \max\{\delta_j(X_j - z_2), 0\} \middle| X_j \le \phi_j(b)\right] \\ & + \delta_i \Pr\left[X_j > z_2, X_j - z_1 < \delta_j(X_J - z_2) \middle| X_j \le \phi_j(b)\right] \\ & + \delta_i x_i \Pr\left[X_j - z_1 < \delta_j(X_j - z_2), X_j < z_2 \middle| X_j \le \phi_j(b)\right] \end{aligned}$$

After some algebraic manipulations, we get

$$\begin{aligned} & z_1\left[1 - G\left(\max\left\{z_1, \frac{z_1 - \delta_j z_2}{1 - \delta_j}\right\}\right)\right] + \delta_i z_2\left[G\left(\frac{z_1 - \delta_j z_2}{1 - \delta_j}\right) - G(z_2)\right] \\ & + \delta_i x_i G\left(\min\left\{z_2, \frac{z_1 - \delta_j z_2}{1 - \delta_j}\right\}\right) \end{aligned} \tag{1}$$

The problem of seller is to find a sequence of prices (z_1, z_2) that maximize (1). However, we can greatly simplify the problem if we impose a constraint of monotonicity on the dynamics of prices. For that, we make some considerations. First, we notice that the seller never chooses prices such that $\min\{z_1, z_2\} > \phi_j(b)$, otherwise the buyer will refuse any offer. Second, at the description of bargaining process, the itens 1) and 4) allow us consider the seller as a monopolist of durable good. Thus, just like shown by Gul, Sonnenschein and Wilson (1986), our model of bargaining is similar to a dynamic monopoly in which the equilibrium prescribes a nonincreasing sequence of prices. In short, we can assume, ex ante, equilibrium prices meets the Coase Conjecture.

Given the above arguments, the problem of the seller can be formally described as

$$\max_{z_1,z_2} z_1\left[1-G\left(\frac{z_1-\delta_j z_2}{1-\delta_j}\right)\right]+\delta_i z_2\left[G\left(\frac{z_1-\delta_j z_2}{1-\delta_j}\right)-G(z_2)\right]+\delta_i x_i G(z_2) \tag{2}$$
$$\textit{such that}$$
$$0\le z_2 \le z_1 \le \phi_j(b)$$

Let the virtual value of *G*, denoted by φ, as

$$\varphi(z)=z-\frac{1-G(Z)}{g(z)}$$

The Theorem 2 below characterizes the solution of (2).

Theorem 2 *Suppose that the ϕ is an increasing function*[4]*. We have*

a) *If* $\delta_i \le \delta_j$ *then the equilibrium prices at the bargaining stage are* $0<\tilde{z}_1=\tilde{z}_2<\phi_j(b)$;

b) If $\delta_i > \delta_j$ *then the equilibrium prices at the bargaining stage are* $0<z_2^*<z_1^*<\phi_j(b)$*;*

Proof The proof is similar to proof of Theorem 1 in Sobel and Takahashi (1983) and it is omitted.

Let $(z_1(x_i),z_2(x_i))$ the solution of (2). On the side of buyer, if him value is x_j and he offer a bid *b*, then the discounted expected payoff at the beginning of bargaining stage is given by

$$B(x_j,b)=\mathrm{E}_{X_i}\left[(x_j-z_1(X_i))\mathbf{1}_{\{x_j\ge\max\{z_1(X_i),S(X_i)\}\}}+\delta_j(x_j-z_2(X_i))\mathbf{1}_{\{z_2(X_i)\le x_j\le S(X_i)\}}\middle| X_i\ge\phi_i(b)\right] \tag{3}$$

where $S(X_i)=(z_1(X_i)-\delta_j z_2(X_i))/(1-\delta_j)$. Under the Coasian Conjecture and assume that $S(\cdot)$ is invertible[5], (3) can be rewriting as

$$B(x_j,b)=\delta_j\int_{\phi_i(b)}^{\omega_i}(x_j-z_2(x_i))\frac{f_i(x_i)}{1-F(\phi_i(x_i))}dx_i+(1-\delta_j)\int_{\phi_i(b)}^{S^{-1}(x_j)}(x_j-S(x_i))\frac{f_i(x_i)}{1-F(\phi_i(x_i))}dx_i$$

[4] This means that *G* is Myerson-regular. See Myerson (1981).
[5] This assumption is critical. However, it is possible to show that if *G* is Myerson-regular and log-concave then *S* is strictly increasing, for all values of δ_i and δ_j.

2.2 Auction and the Main Result

Let $R(b, x_i)$ the value function from problem (2). The problem of seller at the auction stage is find a bid *b* that solves

$$\max_b \delta_i R(b, x_i) - bF_j(\phi_j(b)) \tag{4}$$

Using the Envelope Theorem in $R(b, x_i)$, the first-order conditions of (4) results

$$\left(\delta_i z_1(x_i, b) - b\right) f_j(\phi_j(b))\phi_j^{'}(b) = F_j(\phi_j(b)) \tag{5}$$

If in equilibrium $x_i = \phi_i(b)$ and letting $z_t(\phi_i(b), b) = z_t(b)$ for *t* = 1,2 then (5) becomes a differential equation as below

$$\frac{d \ln F_j(\phi_j(b))}{db} = \frac{1}{\delta_i z_1(b) - b} \tag{6}$$

On other hand the problem of buyer at the auction stage is find a bid *b* that solves

$$\max_b (x_j - b) F_i(\phi_i(b)) + \delta_j \int_{\phi_i(b)}^{\omega_i} B(b, x_i) f_i(x_i) dx_i \tag{7}$$

The first order conditions of (7) is given by

$$\begin{aligned}&(x_j - b) f_i(\phi_i(b))\phi_i^{'}(b) - F_i(\phi_i(b)) - \delta_j^2 \left(x_j - z_2(\phi_i(b))\right) f_i(\phi_i(b))\phi_i^{'}(b) \\ &- \delta_j(1-\delta_j)\left(x_j - S(\phi_i(b))\right) f_i(\phi_i(b))\phi_i^{'}(b) = 0\end{aligned} \tag{8}$$

If in equilibrium $x_j = \phi_j(b)$ then the equation (8) results at the differential equation

$$\frac{d \ln F_i(\phi_i(b))}{db} = \frac{1}{(1-\delta_j)\phi_j(b) + \delta_j z_1(b) - b} \tag{9}$$

Therefore, to find the equilibrium bids at the auction with resale require solves the differential equation system given by (6) and (9) for any $b \in [0, \bar{b}]$ with the boundary conditions $\phi_i(0) = \phi_j(0) = 0$ and $\phi_\lambda(\bar{b}) = \omega_\lambda$ for $\lambda = i, j$.

Notice that if $\delta_i = \delta_j = 1$ then the model results are the same of Hafalir and Krishna (2008), therefore once again resale simmetrizes the auction. On other hand if $\delta_i \leq \delta_j < 1$ then

$$F_i(\phi_i(b)) < F_j(\phi_j(b)) \text{ for any } b \in (0, \bar{b}) \qquad (10)$$

Let $H_\lambda = F_\lambda \circ \phi_\lambda$ for $\lambda = i, j$, so that, H_λ is the equilibrium distribution of bids for the bidder λ. Thus, the inequality in (10) can be rewriting as $H_i(b) \leq H_j(b)$ for any $b \in [0, \bar{b}]$. Therefore H_i first-order stochastically dominates H_j , so that, higher bids are more probable when the bidder is the seller. In the terminology introducing by Maskin and Riley (2000) this means that the seller is the strong bidder at the auction. An immediate corollary is that $\mathrm{E}_i[b] \geq \mathrm{E}_j[b]$ and therefore when the seller is more patient than the buyer her bids are, at mean, bigger than the bids of buyer.

At the same way we can show that if $\delta_j < \delta_i$ then $H_j(b) \leq H_i(b)$ for any $b \in [0, \bar{b}]$, and the conclusions are reversed: when the buyer is more patient than the seller he/she will be the strong bidder at the auction. These results are summarized in the Theorem 3 below.

Theorem 3 *According to the terminology introducing by Maskin and Riley (2000), the bidder on the asymmetric first-price auction with resale that has the lower discount rate will be the strong participant on equilibrium. On other hand if the discount rates are equals to unity, then resale symmetrize the auction.*

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