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ON THE FORWARD-LOOKING BEHAVIOR OF BRAZILIAN CENTRAL BANK REGARDING THE TOTAL RESERVES ♦

Paulo Rogério Faustino Matos ♣
Graduate Program in Economics - CAEN/UFC

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ABSTRACT

We add to the discussion about a puzzle in international finance: the excess reserves in emerging countries, by proposing a methodological innovation aiming to model the optimal time path of the amount of international reserves in Brazil. Based on previous evidences for this economy, we extend the microfundamented benchmark buffer stock model proposed by Frenkel and Jovanovic (1981), assuming a time-varying forward-looking behavior of Brazilian monetary authority. We perform an empirical exercise, by applying this model for a weekly database from June 02, 2003 to December 28, 2015, taking into account multiple break dates as thresholds useful to identify five subperiods. According to our main findings, net public sector debt to GPD ratio, SELIC rate and GDP growth are some of the macroeconomic variable whose change in expectations seems to be more relevant to influence level of reserves. Our forward-looking extension provides a gain of explanatory power in relation to the benchmark model, mainly in the period of greatest turbulence from April 2007 to June 2009. We believe that our framework can be useful to support Brazilian policy makers decisions about driving the stock of international reserves.

Keywords: Brazilian international reserves; Central Bank of Brazil; Time-varying forward-looking behavior; Microfundamented buffer stock model.

JEL Codes: G12; G15

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♣ Address correspondence to Paulo Matos, Graduate Program in Economics at Federal University of Ceará, Fortaleza – Ceará, Brazil. E-mail: paulomatos@caen.ufc.br.

1 INTRODUCTION

Emerging economies are more likely to experience periods of turmoil due to economic crises not only local but also global or punctual in other economies. According to Fidrmuc and Korhonend (2010) and Ozkan and Unsal (2012), a possible reason is the robust empirical evidence of greater intensity of contagion effects among emerging economies. Developed and emerging countries still differ with respect to sensitivity to the crisis effects. For instance, Laeven and Valencia (2013) show that banking crises impose costs on the fiscal side of the developed economies of 3.7 % of Gross Domestic Product (GDP), while in developing countries this cost is close to 11.5 %.

This brief but fundamented context suggests that the relationship between developed or developing economies and the crises is quite different. Observing the related literature about behavior in crises, one can evidence that there is disagreement on the exact combination of foreign exchange, credit, monetary and fiscal policy to be used against the crisis. This issue depends on the degree of openness of the economy, level of development, maturity of its financial system, among other political, social and economic variables.

According to this literature, the only consensus seems to be the relevance of having a special concern for the protection mechanisms of developing economies financial systems by policy makers.

In this discussion, if we limit the analysis only on preventive protection measures, research object of this paper, Feldstein (1999) argues that central banks in emerging economies have smaller and more limited access to external financing sources and even under worse conditions compared to developed economies. Therefore, emerging economies should not rely on the possibility of loans from international organs, such as International Monetary Fund (IMF), or partial economic reforms. In practice, the most promising researching routes point to the priority of emerging nations in increasing the liquidity of international assets, reducing the short-term external liabilities, implementing measures for credit leverage and increasing international reserves. [1]

Specifically on the management of those reserves, an extensive literature – from Heller (1966) to recent contributions, as Calvo, Izquierdo and Loo-Kung (2012) – suggests that monetary authority should accumulate reserves to facilitate international trade, protect the national currency from speculative attacks and protect the financial system from capital scape. This literature also mentions that the monetary authority intervenes in the foreign exchange market aiming to practice a stock buffer, i.e., protecting of society from the demand or supply variations able to change significantly the prices of the asset in question. In sum, the stock of

[1] See Rodrik and Velasco (1999) about the short-term debt over the absorption of recent crises impacts. The relevance of credit leverage can be seen in the real estate crisis in the USA.

international reserves of a country – the sum of any kind of reserve funds that can be passed between the central banks of different countries, held by monetary authority as a guarantee for payment obligations – consists of a public asset that can be used to ensure continuity of economic activity and preserve financial stability.

In this scenario, there is a puzzle: the excess reserves in emerging countries, even considering that emerging and developed countries have different optimal solutions against crisis. Pina (2015) reports a clear divergence in the ratio international reserves to GDP comparing emerging and developed countries. This author shows that at the beginning of the sample in 1987, the unweighted cross-country average of this ratio for 24 developed countries was close to 10%, while for 154 emerging this average was close to 11%. Thenceforth, one can evidence paths with downward trend for developed economies and growing trend for emerging ones. At the end of 2007, this ratio reaches a level of 7% in the major world economies, while emerging accumulate on average over 25 % of its GDP in reserves.

Is this evidence empirical support enough for claiming to be a puzzle? Is this a puzzle for many emerging economies, or only some economies have excess reserves without justification? What is the cost of an excessive conservatism? Is this the puzzle a consequence of the absence of an optimal path reserves model yet to be written?

Faced with these questions, we add to this debate by proposing a methodological innovation aiming to model the optimal time path of the amount of international reserves in Brazil, which enables us to answer whether there is an excess of conservatism or not and thus whether there is a puzzle in Brazil or not.

More specifically, we propose an innovative and specific framework able to model total reserves in Brazil in a higher frequency, weekly, that: i) accommodates previous empirical evidences about relevance or not of main sources of contagion effects, ii) enables a dynamic behavior of Central Bank of Brazil and iii) incorporates the forward-looking behavior of this monetary authority.

A common feature of existing frameworks for modeling the reserves is the premise that keeping an intertemporal trajectory offers benefits such as protection, but necessarily implies social and financial costs associated with exchange rate management policies, as argued by Rodrik (2006). It is important to mention that according to studies as Durdu, Mendoza and Terrone (2009), Alfaro and Kanczuk (2009), Jeaane and Rancière (2011) and Pina (2015), the use of reserves is able to smooth the consumption of society and levels investments. Therefore, the ideal policy should not be accumulating reserves excessively, because there is a theoretical level that should be seen as a benchmark, i.e., it is able to optimize the cost–benefit of such level of reserves. [2]

[2] There are theoretical and empirical paper modeling monetary policy that take into account the level of reserves as exogenous. See Burnside, Eichenbaum and Rebelo (2001) and Kumhof, Nunes and Yakadina (2010).

This is exactly the issue we want to address: suggesting a forward-looking approach specific to Brazilian economy able to identify this optimal path of reserves for this country. We follow conceptually Heller (1966) and in theoretical terms, we are aligned to Ben–Bassat and Gottlieb (1992) and Chakravarty (2009), aiming to propose an innovative framework based on idiosyncratic extensions to the model developed by Frenkel and Jovanovic (1981), entitled buffer stock model. This model considers the associated adjustments to the exchange rate and monetary policy besides the opportunity cost of holding such volume of reserves. In terms of application, we follow Ford and Huang (1994), Ramachandran (2004), Jeanne (2007) and Pina (2015), related studies that analyze reserves in emerging economies. Aiming to compare our results obtained for Brazil, we should mention empirical related studies: Cavalcanti and Vonbun (2008), Matos *et al*. (2016b) and Matos and Reis (2016).

Our methodological innovation in relation to previous studies is the use of this buffer framework stock, but taking into account that Central Bank of Brazil has a time varying forward-looking behavior concerning the stock of international reserves. We must mention the evidence reported in Matos and Andrade Neto (2015) about the forward-looking behavior in the SELIC rate decision by the Monetary Policy Committee of the Central Bank of Brazil (COPOM). Aligned to Chakravarty (2009), we also add state variables to the model of Frenkel and Jovanovic (1981), but not ad hoc contemporary variables. We incorporate with statistical criterion expectations series of key macroeconomic variables. Since this is our main novelty, we implement carefully this procedure of insertion. We use the expectations series for the economic variables monitored by Market Report Focus of the Central Bank of Brazil (BCB) that reflect the market perception for the macroeconomic performance of the country. The best specified version of the model is based on Akaike (1974) criteria information.

In our specific case, the exercise applies the model proposed here for a weekly database from June 02, 2003 to December 28, 2015. Because this is a long time series of reserves, in our exercise we are very carefully concerning multiple structural breaks, so that we estimate our model to each one of the five regimes characterized by four break dates identified. This procedure allows us to identify different behavior of Brazilian monetary authority over time, i.e., we can evidence a dynamic reaction over time of Central Bank of Brazil regarding changes of expectations in most relevant macroeconomic variables.

The relevance of our paper is first due to the specific innovative methodological contribution to Brazilian economy. Second, our empirical exercise is useful to suggest an optimal trajectory, which allows us to characterize whether there is an excess or lack of reserves, then supporting the decision of the Central Bank of Brazil to adopt a policy more or less conservative. This optimal theoretical path can also be used to support the decision about using these reserves for purposes other than the protection of the country, as suggested by some federal authorities. In short, without a reliable optimal path as a result of an exercise

based on a statistically refined model, the policy makers should not make decisions about the composition of these reserves, nor using these reserves for other purposes.

To summarize some of our main findings, net public sector debt to GPD ratio is the macroeconomic variable whose change in expectations seems to be more relevant, to be significant in all periods, except for the period from April 2005 to March 2007. The results suggest that expected increases in government spending tend intuitively to affect the reserves in a more conservative direction by the Central Bank of Brazil. We also emphasize that bad news associated with the Brazilian GDP growth suggest that the monetary authority should increase the amount of reserves.

This paper is divided into five sections. Section 2 analyzes the evolution of reserves in Brazil and other economies. The third section details the methodology, and the empirical exercise reported in section 4. The fifth section presents the final considerations.

2 AN OVERVIEW CONCERNING TOTAL RESERVES IN BRAZIL

Under the assumption that there is an optimal theoretical level to be treated as a benchmark dealing with the cost–benefit of such level of reserves, the theoretical and empirical literature has evolved to identify, quantitatively or qualitatively, if there is an excessive conservatism or an adoption of a policy unable to protect financial system.

In one of the first studies in this area, Heller (1966) analyzes historical series that show that Brazil had a level of reserves classified only as "satisfactory" in 1963, with US$ 318 million, while the ideal would be US$ 570 million. Consequently, Brazilian rating reached out 0.56, calculated as the ratio between the actual level and the great. In this same period, China and India ratings reached out 1.67 and 0.48, respectively, for instance.

Another useful measure able to support a scenario of excess or not of conservatism works as a role of thumb: total reserves should cover at least three months' worth of imports. Observing this metric for Brazil in Matos and Reis (2016), it ranges from 2.6 to 3.7 from 2004 to 2006. From 2007 to 2014, this ratio ranges from 3.5 to 6.9 and in 2015 it has an upward trend, reaching out more than 9.1 months of import in December 2015. This ratio is the highest if one compares Brazil with other major South American economies: 6.9 in Peru followed by 5.4 in Argentina.

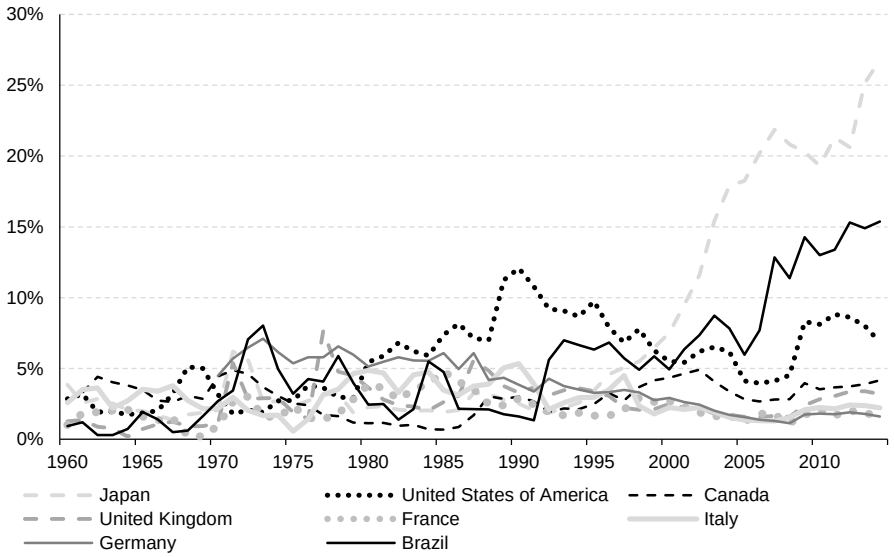
Some papers use to weight the amount of reserves by the respective GDP aiming to establish a comparison. The related evidences suggest that during the period from 1960s to the 1990s, both emerging and developed seemed to converge in the conduct of policy of accumulation of reserves characterized by trajectories without growth trends and with mild fluctuations. During this period, 10% worked as an upper bound for reserves to GDP ratio for most economies. For details of such data, see Lane and Milesi–Ferretti (2007).

The divergence reported in this literature is recent. One can observe this pattern in papers such as Pina (2015) that uses samples with hundreds of countries.

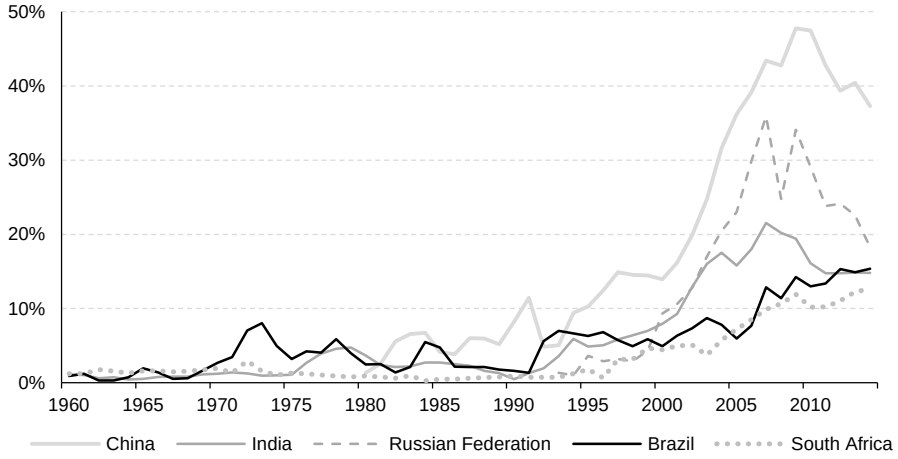
Taking into account a smaller, however representative sample of developed and emerging economies, Figure 1 shows the annual series from 1960 to 2014 of reserves in Brazil compared to more developed economies, entitled G7 (Figure 1.a) and compared to the other BRICS countries (Figure 1.b).

Figure 1. - Total reserves excluding gold (% of GDP). Data source: FRED

1.a. Annually total reserves excluding gold (% of GDP) for Brazil and G7 economies



1.b. Annually total reserves excluding gold (% of GDP) for Brazil and BRICS economies



The comparison with the G7 countries suggests that Brazil had reserves to GDP ratio with an order of magnitude close to other countries, with a visually sharper oscillation, until

the early 1990s. Thenceforward, Brazilian path takes off the other G7 paths, breaking the barrier of 5% and situating below only the reserves to GDP of USA, temporarily.

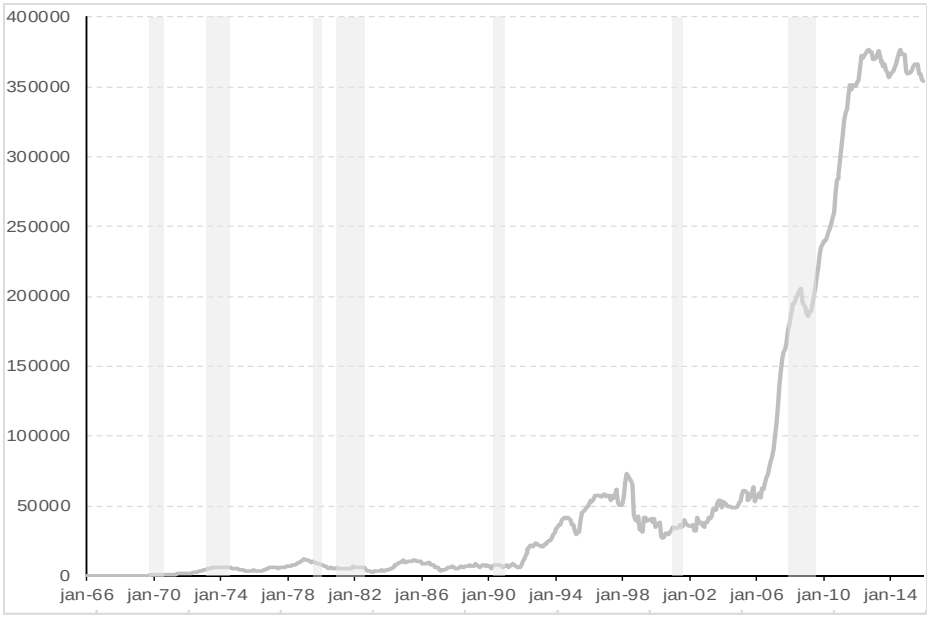
In the following decade, the policy of accumulation of reserves is clearly more conservative, with a very positive trend in the path, reaching out in 2014 the level of 15%. Only in Italy, we have a reserve to GDP ratio higher, about 27%. Observing all other G7 countries, this ratio ranges from 1.6% in Germany to 6.9% in USA.

The comparison of Brazil with Russia, India, China and South Africa, the other emerging economies composing BRICS, suggests that the trajectories over these fifty-five years have been close until the 1980s. Thenceforth, China has emerged as an outlier, with a stock reaching out almost 50% of GDP in 2010, followed by Russia.

These economies have similar behaviors as well, if we look at the data from 2009 onwards, characterized by a strong reduction in the relative reserve level. Also according to Figure 1.b, Brazil's trajectory over the last fifteen years is quite close to South Africa, both presenting a ratio at the end of 2014 close to that reported by India.

For a more specific analysis of Brazil, associated with an international situation, Figure 2 shows the monthly trajectory of reserves (US$ million), from January 1966 to December 2015.

Figure 2 - Total reserves excluding gold for Brazil (US$ million). Data source: FRED



[a] Monthly total reserves excluding gold (US$ million) for Brazil during normal periods (white areas) and crisis periods (gray areas) according to NBER.

A first aspect is that, although there was a visible “jump” of the absolute values of reserves in the last decade, 2006–2015, the relative increase in this period was close 530%,

lower than the growth of 850% observed in the first decade reported in Figure 2, 1966-1975. During this decade, the level rose from US$ 420 million to US$ 3.98 billion. One can mention that it is necessary to consider that during the first decade of analysis, GDP at current prices rose about 350%, while in the last decade the increase was just over 100%.

Regarding the intermediate decades, from 1976 to 1985, the growth rates were respectively 180% and 460%. In the decade between 1996 and 2005, there was a stagnation, if we consider the levels of January 1996 and December 2005, US$ 51.5 billion and $53.2 billion respectively. In this period, the increase was close to 3%.

Figure 2 also allows us to monitor reserves in periods characterized as crises according to The National Bureau of Economic Research (NBER). During the eight periods of crisis highlighted, only the first crisis, between December 1969 and November 1970, and during the last crisis, from December 2007 to June 2009, there were increases that are more significant. In the first crisis, the reserves increased from US$ 611 million to over US$ 1.1 billion, an increase close to 80%, while in the last crisis, the increase was only 11.7%.

3 METHODOLOGY

3.1. Brief history of Theoretical Evolution

Observing the history of the theoretical frameworks and their empirical applications, one can point an extensive literature, so that the discussion of how adequate are the models for each situation or economy is not consensual. The unique common aspect of all these approaches seems to be the foundation of any cost–benefit analysis based on macroeconomic variables, following Krugman (1979), or financial variables, as foreign exchange, according to Calvo and Reinhart (2002), or political–institutional variables, aligned to Cheung and Ito (2009).

In this context, possibly Heller (1966) has been a pioneer in promoting the debate on this issue, combining the optimal level of reserves to a protective instrument (buffers) to smooth balance of payments imbalances and incorporating variables linked to the cost of adjustment and opportunity cost of reserves.

Some related contributions at this time are Kenen and Yudin (1965) and Machlup (1966), which introduced the use of econometric techniques. After the Bretton Woods system in the early 1970s, it is evident an interest in the topic again, with contributions from Heller and Kahn (1978), Edwards (1985), Frenkel (1983), Lizondo and Mathieson (1987), among others. Another moment of higher intensity of this literature only arises because of the crisis of the European Monetary System and the crisis "Asian tigers" in the 1990s.

In a didactic way, it is usual to classify the frameworks as first, second and third generations. The first generation is exemplified by Krugman (1979) and Flood and Garber

(1984), which emphasized the role of reserves as a crisis postponement tool. These papers were followed by models of currency crises second generation, instituted by Obstfeld (1994), while the third generation was inaugurated by Ben–Bassat and Gottlieb (1992) and Furman and Stiglitz (1998).

3.2. The Benchmark Buffer Stock Model of Frenkel and Jovanovic (1981)

We follow one of the most promising routes of this third generation, the Frenkel and Jovanovic (1981) buffer stock model, which takes into account adjustments due to monetary policies besides the opportunity cost of keeping such volume of international reserves. Frenkel and Jovanovic (1981) define reserve movements in continuous time period following a Weiner process given by:

$$dR_{i,t} = -\mu_i . dt + \sigma_i . dW_{t,} \tag{1}$$

where $R_{i,t}$ denotes reserves held by economy i in time t, μ_i and σ_i are the respective mean and standard deviation of the Weiner increment in reserves and W_t means a standard Weiner process with zero mean and variance t. At each point of time, the distribution of reserve holdings $R_{i,t}$ can be characterized by:

$$R_{i,t} = R^*_{i,t} - \mu_i . t + \sigma_i . W_t \tag{2}$$

In the relation (2), $R^*_{i,t}$ is the optimal stock of international reserves, which is obtained when one minimizes the cost of adjustment – incurred once reserves reach an undesirable lower bound – and foregone earnings on reserve holdings. The final results of this simple, but very useful approach is a benchmark reserve demand equation in most empirical papers, given by:

$$ln\left(R^*_{i,t}\right) = \beta_0 + \beta_1\, ln\left(\sigma_{i,t}\right) + \beta_2\, ln\left(r_{i,t}\right) + \epsilon_{i,t} \tag{3}$$

Regression (3) suggests that the optimal level of reserves (in log), $ln\left(R^{b,*}_{i,t}\right)$, depends linearly on standard deviation of change in reserves (in log), given by $ln\left(\sigma_{i,t}\right)$, and on opportunity cost of holding reserves (in log), given by $ln\left(r_{i,t}\right)$. In this relation, $\epsilon_{i,t}$ means the residual and one could expect theoretically that $\beta_1 = 0.5$ and $\beta_2 = -0.25$.

3.3. Some Extensions of the Buffer Stock Model

In sum, based on the assumption that the balance of payments of economy i is in equilibrium and that reserves follow a stochastic Wiener process, one can derive the optimal path, $R_{i,t}^{*}$, in order to minimize the macroeconomic adjustment costs and opportunity costs. Assuming that $R_{i,t}^{*}$ follows an approach of second order Taylor, this optimal level can be described as a logarithmic function of the respective conditional volatility of the variation of reserves, $\sigma_{i,t}$, and the opportunity cost associated, $r_{i,t}$.. Although simple, the implementation and possible extensions are not consensual, motivating some routes in this literature.

Although Frenkel and Jovanovic (1981) had the estimated elasticities close to the theoretical predictions of the model, $\beta_1 = 0.5$ and $\beta_2 = -0.25$, many studies, as Flood and Marion (2002) and Ramachandran (2004), for instance, have obtained different values of the elasticities. Chakravarty (2009) argues that the reason is these estimates are highly sensitive to the proxy used to represent opportunity cost, model specification, estimation methods and additional variables included in the original equation.

The most promising route following Frenkel and Jovanovic (1981) suggests extensions to their benchmark framework in the sense of incorporating some idiosyncratic additional variables, which are important in the determination of the level of reserves for specific emerging economies. Chakravarty (2009), for instance, adds a scale variable, $y_{i,t}$, due to the positive correlation between reserve holdings and the size of international transactions. In this empirical study for India, the author also includes a variable representing the flexibility of the exchange rate, $e_{i,t}$, because the exchange rate has become market determined in this economy.

In this case, the modified buffer stock proposed by Chakravarty (2009) can be written as:

$$ln(R_{i,t}^{*}) = \beta_0 + \beta_1\, ln(\sigma_{i,t}) + \beta_2\, ln(r_{i,t}) + \beta_3\, ln(y_{i,t}) + \beta_4\, ln(e_{i,t}) + \epsilon_{i,t} \quad (4)$$

3.4. Our extension of the buffer stock model for Brazilian reserves

We follow Chakravarty (2009) by proposing an extension that applies to the reality of a specific emerging economy. However, we take into account for the idiosyncrasies and empirical evidences reported for the Brazilian economy.

First, we need to know whether this optimal trajectory of reserves in Brazil depends or not on contagion and financial integration between Brazil and other emerging economies. Regarding this issue, one can see an extensive literature reporting these effects between BRICS economies, as in Misra and Mahakud (2009), Chittedi (2010) and Matos *et al*. (2016a). In this context, Matos *et al*. (2016b) incorporate contagion and integration effects on reserves in

BRIC by estimating a joint buffer stock benchmark model for these economies. According to them for the period from 1997 to 2013, Brazil is the unique whose reserves are solely dependent on the own series of spread and volatility.

Taking into account those cross-effects between major South American economies, although Matos *et al.* (2014) evidence short- and long-term relationships between Argentina, Brazil, Chile, Colombia and Peru, according to Matos and Reis (2016) only Brazilian reserves seems to be independent of other South American series of imports, spread and volatility. Therefore, our forward-looking model is based on this evidence, i.e., we dismiss cross-effects due to contagion between BRIC or South American economies, main emerging partners of Brazil.

Secondly, our extension accommodates another recent empirical finding about Brazil. According to Matos and Andrade Neto (2015), one can evidence a forward-looking behavior of the conduct policy of the interest rate by the Monetary Policy Committee of the Central Bank of Brazil (COPOM). They follow methodologically a statistical framework developed by Engle and Russel (1998) and used by Hamilton and Jordà (2002) in studying the decisions of the Federal Reserve of the United States. Those authors find that for the period from January 2002 to July 2010 with a weekly frequency COPOM has a forward-looking behavior, holding onto the expectations of GDP and Brazilian official inflation.

The third issue concerns the relevance to give freedom to the model so that it allows the optimal path reserves to react differently over time given the opportunity cost, the conditional volatility and changes in expectations of main macroeconomic variables. We do it by identifying multiple break dates based on tests that compare information criteria for 0 to M globally determined breaks.

To summarize, we propose a model for reserves for Brazil taking into account only domestic variables, incorporating a time-varying forward-looking behavior of the Central Bank.

Our extension can be described as follows:

$$ln\left(R_{i,t}^{*}\right) = \beta_0 + \beta_1\, ln\left(\sigma_{i,t}\right) + \beta_2\, ln\left(r_{i,t}\right) + + \sum_{s=1}^{13} \varphi_s^i x_{s,t} + \varepsilon_{i,t} \tag{5}$$

$$\varepsilon_t^i | \Psi_{t-1} \sim N(0, {\sigma_t^i}^2) \tag{6}$$

$${\sigma_t^i}^2 = \theta_0 + \sum_{l=1}^{q} \theta_l\, {\varepsilon_{t-l}^i}^2 + \sum_{j=1}^{p} \gamma_j\, {\sigma_{t-j}^i}^2 + \vartheta_t^i \tag{7}$$

Regression (5) suggests that the optimal level of reserves (in log) depends linearly on standard deviation of change in reserves (in log) and on opportunity cost of holding reserves (in log). Also in this regression, φ_s^i measures the reaction of the Brazilian reserves to changes of expectation of variable s in question, expressed by $x_{s,t}$.

More specifically, we follow most of the literature by determining the opportunity cost as the difference between the cost of fund-raising by the Brazilian government and the interest rates earned on the application of reserves in US government bonds. [3] In Brazil, empirical studies commonly use of the log of the ratio between the gross return on SELIC rate and the gross return on 90-day or 1-year Treasury Bill. This is a proxy for the cost to society when the level of reserves is above the optimal level, and should be set down in such a situation.

The second step is the adoption of a framework for modeling the volatility.

Engle (1982) suggests a conditional variance heteroskedastic as a linear function of the square of past innovations, giving rise to the famous framework entitled Autoregressive Conditional Heteroscedasticity (ARCH). Aiming to get a more parsimonious framework, no major problems with signal parameters and to allow both a long memory and a more flexible lag structure, we follow the extent suggested in Bollerslev (1986) entitled Generalized ARCH (GARCH).

Here, we follow West and Cho (1995), who show that for short time horizons, exercises following the GARCH family of frameworks are more accurate and appropriate to predict volatility, than a constant standard deviation or even compared to other frameworks of conditional volatility. A recent application of this very interesting framework for Brazil and other Latin American countries is Hegerty (2014).

Concerning the GARCH model, ε_t^i is the demeaned series of reserves, which follows a normal, and whose conditional variance ${\sigma_t^i}^2$ is expressed by equation (7). About this equation of variance, as usual in the GARCH specification, we have that $p \geq 0,\ q > 0,\ \theta_0 > 0, \theta_l \geq 0,$ $\gamma_j \geq 0,$ e $0 \leq \sum_{l=1,j=1}^{\max(p,q)} \theta + \gamma_j < 1$. As reported in this literature, low-order GARCH are used in most applications Therefore, we have to find the best specification for the GARCH, i.e., defining how many lags, p and q, respectively. ϑ_t^i is the residual of the variance equation.

In the third step, we have our innovation key by incorporating the dependence of reserves in relation to the change in expectation of macroeconomic variables, which are monitored and reported on Market Report - Focus, published weekly by the Central Bank of Brazil (BCB).

Our extension based on identifying which forward-looking explanatory variables must be incorporated into the final specification framework is a non-trivial step, according to

[3] During the period from June 2003 to December 2015, more than 95% of Brazilian reserves is given by foreign government bonds and currencies.

Tsay (2005), because we have thirteen forward-looking variables and thus, more than 8,000 possible combinations with them. Here, we get the forward-looking buffer stock model by means of recursive programming routines, reporting here the results based on the specification from those thousands with better Akaike criterion information.

This procedure is useful to model conditional volatility of Brazilian sectorial indices, according to Matos *et al*. (2016c)

4 EMPIRICAL EXERCISE

4.1. Endogenous Variables: Data, Summary Statistics and Multiple Breaks

Following buffer stock literature, the endogenous variables in our model, is the total reserves for Brazilian economy in log.

Most of related empirical applications use monthly or quarterly data, while we work with weekly data instead of daily, because of the availability of expectations series in the report Focus of the Central Bank of Brazil: this report is always disclosed to the financial market on the first working day of each week. Given our purposes to measure the effects of expectations, we use the daily series for total reserves (cash concept), available from September 01, 1998 to nowadays, in Central Bank of Brazil datacenter.

We have to work with a shorter series, from June 2, 2003 to December 28, 2015, because of the availability of some exogenous variables, such expectation series, which are more recent and date from 2002 and 2003. Throughout these 657 weeks, total reserves, including gold, range from US$ 43.3 billion to US$ 379.7 billion.[4] Considering only initial and final values, there is an increase of 722%, which corresponds to a geometric average weekly growth rate of 0.32%. The average value of these reserves was US$ 223.7 billion, with a standard deviation of US$ 128.5 billion.

Since we need to deal with stationary series, we identify the endogenous break dates estimated by multiple break tests applied to reserves in log.

According to Table 1, both tests suggest the four breaks and the estimated break dates are also the same. We can see that except for the latter regime, other have similar length, close to two years. The penultimate break coincides with the beginning of the recent mortgage crisis in United States of America.

[4] It is common in this literature using the International Financial Statistics definition of total Reserves of the monetary authorities minus gold, which is excluded because there is some doubt if central banks consider gold as liquid as foreign currency holdings. In Brazil, during the period from June 2003 to December 2015, the composition of reserves is given by: 81.7% in foreign government bonds, 14.3% in foreign currencies, 0.7% in gold, 0.7% in special drawing rights, 0.4% in reserves position in IMF and 2.2% in other assets.

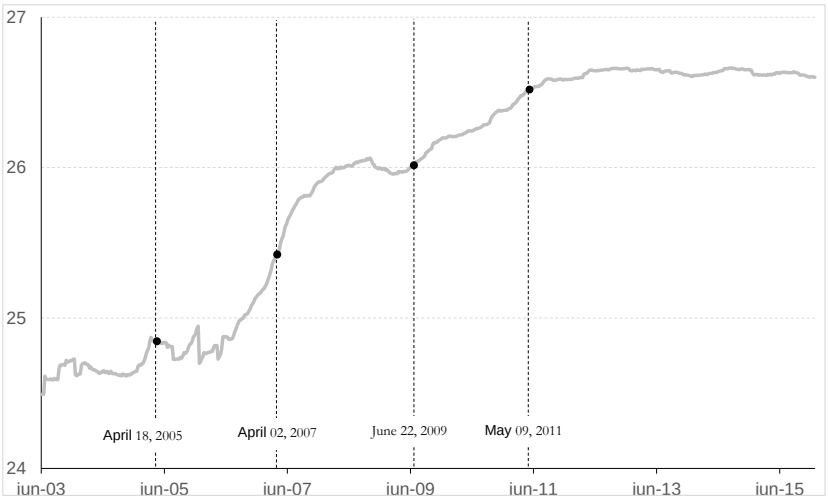
Table 1 - Multiple break tests applied to total reserves (in log) for Brazil during the period from June 09, 2003 to December 28, 2015

	Compare information criteria for 0 to M globally determined breaks [b]		Bai-Perron tests of 1 to M globally determined breaks [c]	
Breaks	Schwarz Criterion	LWZ Criterion	Scaled F-statistic	Weighted F-statistic
0	-0.494	-0.479		
1	-2.527	-2.483	28.312	28.312
2	-3.731	-3.657	547.875	651.076
3	-3.999	-3.896	453.316	652.593
4	-4.287	-4.155	1227.182	2110.064
5	-4.270	-4.108	900.684	1976.437
	Estimated break dates:		Estimated break dates:	
	April 18, 2005		April 18, 2005	
	April 02, 2007		April 02, 2007	
	June 2, 2009		June 2, 2009	
	May 09, 2011		May 09, 2011	

[a] For both methods, quadratic-Spectral kernel, Andrews bandwidth) assuming common data distribution. [b] Minimum information criterion values displayed with shading. [c] Maximum statistic values displayed with shading. Significance based on Bai-Perron (2003) critical values.

In Figure 3, we show the total reserves in log and respective breaks identified.

Figure 3 - Weekly total reserves (in log) for Brazil and respective breaks from June 02, 2003 to December 28, 2015. Data source: Central Bank of Brazil



4.2. Exogenous Variables of Benchmark Buffer Stock Model

We have shown in subsection 3.2 that based on the benchmark buffer stock model, the optimal level of reserves (in log), depends linearly on standard deviation of change in reserves (in log) and on opportunity cost of holding reserves (in log). First, we follow a method commonly used in the literature, namely the fiscal cost of sterilizing reserves, by computing this opportunity

cost of reserves the spread given by the differential between the Brazilian government policy rate (SELIC) and the yield on short term U.S. government bonds (1-year Treasury bill). Observing these rates per year during the period of our analysis, SELIC ranges from 7.11% to 26.27%, while Treasury bill ranges from 0.01% to 5.05%, suggesting a spread significantly higher than usually observed in other economies, even if we compare with emerging ones.

According to Matos and Reis (2016), considering the period from January 2004 to December 2015, the highest spreads reported in South American economies are 22.65% per year in December 2015 in Argentina. In Chile, Colombia, and Peru, spreads are lower than 9% per year. Considering BRIC economies from January 1997 to December 2013, Matos *et al*. (2016b) report that only Russian spread reaches out higher values than Brazil: more than 85% per year in 1998.

Second, given the necessity to measure the conditional volatility in the management of international reserves, a wide range of macroeconomic, econometric and financial frameworks are suggested and proved to be useful in this modeling. Aiming to accommodate anomalies and stylized facts, the empirical literature based on buffer stock commonly makes use of the techniques that make up the family Autoregressive Conditional Heteroskedasticity (ARCH) and its extensions. Here, we follow Dominguez (1998), choosing the most appropriate specification among parsimonious models.

More precisely, we extract the series of volatility for the reserves change by estimating a GARCH framework for each of the five sub-periods of time identified by multiple break dates test. Aiming at the most parsimonious model, we extract the volatility series among six possible combinations of GARCH, up to two lags of both terms explanatory and besides ARCH combinations with up to two lags of the squared error. We use here the results based on the specification from these eight specifications with better Akaike criterion information. These estimations are reported in Table 3, subsection 4.4, with other estimations. [5]

4.3. Exogenous Variables of Forward-Looking Buffer Stock Model

Table 2 reports for each expectation of macroeconomic variable a simplified notation in the first column and its respective detailed description. Aiming to deal with stationary series, we need to work with change of expectations.

Other relevant detail is the forecast horizon. The series correspond to market expectations of the closing value of the year following the year in which the information is collected at the Central Bank. The question we want to answer is, for instance, if on January 19, 2009, following the disclosure about change of market forecast on GDP growth for 2010 in relation to previous forecast disclosed a week before, this information affects the Brazilian total reserves. We answer this question, analyzing the effects of expectations of all thirteen series of explanatory variables, through the best specification for the forward-looking buffer stock model.

[5] We do not report the series of spread and conditional volatility, but they are available upon request.

Table 2 - Description of expectation of macroeconomic variables [a]

Notation	Detailed description	Maximum	Minimum	Mean	Standard deviation
$\Delta ipca^{exp}$	First difference time series of expectation of anual inflation measured by Price Index to Broader Consumer (known in Brazil, as IPCA)	0.300%	-1.000%	-0.001%	0.088%
$\Delta igpdi^{exp}$	First difference time series of expectation of anual inflation measured by Generalized Price Index - Domestic Availability (known in Brazil, as IGP-DI)	0.500%	-1.010%	-0.003%	0.106%
$\Delta igpm^{exp}$	First difference time series of expectation of anual inflation measured by Generalized Price Index to Market (known in Brazil, as IGP-M)	0.400%	-1.330%	-0.002%	0.110%
Δipc^{exp}	First difference time series of expectation of anual inflation measured by Price Index to Consumer (known in Brazil, as IPC)	0.540%	-0.900%	-0.002%	0.101%
$\Delta forex^{exp}$	Variation (%) time series of expectation of spot foreign exchange rate (R$/US$)	7.813%	-4.569%	0.034%	1.108%
$\Delta selic^{exp}$	First difference time series of expectation of anual SELIC rate	1.000%	-1.500%	-0.002%	0.207%
$\Delta debt^{exp}$	First difference time series of expectation of anual net public sector debt to GDP ratio	1.100%	-2.300%	-0.020%	0.307%
Δgdp^{exp}	First difference time series of expectation of anual GDP grwoth	1.500%	-0.580%	-0.009%	0.120%
Δind^{exp}	First difference time series of expectation of anual industrial production grwoth	1.500%	-3.500%	-0.011%	0.234%
Δcur^{exp}	Variation (%) time series of expectation of current account balance	325.926%	-10.3333%	0.861%	20.122%
$\Delta trade^{exp}$	Variation (%) time series of expectation of balance of trade	180.000%	-75.000%	0.869%	13.530%
Δfdi^{exp}	Variation (%) time series of expectation of foreing direct investment	22.083%	-10.417%	0.236%	2.379%
Δadm^{exp}	First difference time series of expectation of anual inflation measured by administered prices	1.000%	-2.050%	-0.001%	0.169%

[a] Statistics of weekly series of expectations of main macroeconomic variables, during the period from June 09, 2003 to December 28, 2015. Data source: Weekly Focus Report from BM&FBOVESPA

According to descriptive statistics reported in Table 2, some of the expectations show little variability, which may be a limitation to our model, compromising the ability to explain part of the variance of the dependent variable.

4.4. Results

In Table 3, we report all the results of our estimations. First, regarding equation of variance, we estimated all specifications of GARCH up to two lags of residuals and variance, besides considering up to two thresholds, following a TGARCH as proposed by Zakoian (1994). We can see that the best specified version for all subsamples of time does not need to take into account for thresholds. The volatility extracted during the first and the second periods depends only on its own lagged variance and during the third and the fifth periods, the volatility follows an ARCH (1). In the fourth period, a GARCH (1,1) seems to be the best specification. During the second and the fourth periods, we have individual insignificance of some GARCH parameters. For all periods, most of the parameters are significant to 5 %. We do not have problems with explosive trajectories of the conditional risk, or negativity.

Table 3 - Forward-looking buffer stock model estimation [a, b, c]

Estimations	Periods: subsamples based on multiple break tests				
	1st period (97 observations) From August 09, 2003 to April 11, 2005	2nd period (102 observations) From April 18, 2005 to March 26, 2007	3rd period (116 observations) From April 02, 2007 to June 15, 2009	4th period (98 observations) From June 22, 2009 to May 02, 2011	5th period (243 observations) From May 09, 2011 to December 28, 2015
First step: Variance equation based on parsimonious GARCH model with and without threshold					
Constant	7.21e+17*** [0.000]	2.42e+18 [0.140]	1.99e+18*** [0.000]	2.43e+18*** [0.008]	1.46e+18*** [0.000]
ε_{t-1}^2			0.367* [0.076]	0.579* [0.089]	0.238** [0.015]
ε_{t-2}^2					
σ_{t-1}^2	0.423*** [0.000]	-0.382 [0.186]		-0.104 [0.557]	
σ_{t-2}^2		0.633** [0.042]			
Second step: Mean equation based on forward-looking state variable based buffer stock model					
Constant	32.745*** [0.000]	26.714*** [0.000]	29.953*** [0.000]	25.688*** [0.000]	26.720*** [0.000]
Spread in log	-0.760** [0.019]	-5.114** [0.000]	4.722*** [0.000]	11.611*** [0.000]	-0.822** [0.000]
Volatility in log	-0.382*** [0.000]	-0.058* [0.059]	-0.210** [0.002]	-0.023 [0.247]	-0.001 [0.917]
$\Delta ipca^{exp}$					
$\Delta igpdi^{exp}$				0.202** [0.012]	
$\Delta igpm^{exp}$					
Δipc^{exp}					
$\Delta forex^{exp}$				0.014** [0.022]	0.004 [0.128]
$\Delta selic^{exp}$	-0.061** [0.020]		0.106*** [0.008]		
$\Delta debt^{exp}$	0.052*** [0.001]		0.114*** [0.008]	-0.094** [0.025]	0.012** [0.012]
Δgdp^{exp}			-0.234** [0.022]		-0.018** [0.028]
Δind^{exp}			0.248*** [0.004]	0.045* [0.059]	
Δcur^{exp}		0.002* [0.064]	0.001 [0.177]	0.002 [0.120]	
$\Delta trade^{exp}$			-0.007*** [0.003]		
Δfdi^{exp}					
Δadm^{exp}					
Complementary results of mean equation					
Wald test	5.983*** [0.004]	0.002* [0.064]	8.836*** [0.000]	2.835** [0.020]	6.344*** [0.000]
Adjusted R^2	0.167	0.610	0.697	0.830	0.279
Gap of adjusted R	0.019	0.010	0.152	0.040	0.026
Akaike criterion	-2.623	-1.455	-2.087	-2.969	-4.451

[a] Estimation results based on estimation over the period from June 09, 2003 to December 28, 2015. [b] GARCH models estimated through ARCH with normal distribution errors, using the Bollerslev-Wooldridge robust covariance coefficient heteroscedasticity of the residuals. Respective p-values are in the box brackets. [c] We specify the Newey and West (1987) covariance coefficients method, in order to get robustness to heteroskedasticity in mean equation estimations based on OLS method. Respective p-values are in the box brackets. * Indicates significance at 10% level. ** Indicates significance at 5% level. *** Indicates significance at 1% level.

The most relevant results are in the second stage of estimation: the own buffer stock model in its forward-looking extension applied to Brazil.

Regards the explanatory variables that composes the benchmark version, as one could expect, the spread seems to be significant and negative, except for the third and fourth periods, while volatility seems to be counter intuitively negative and significant in the first, second and third periods. When we compare these result to those obtained in the estimation of the benchmark model, there are no significant changes and the parameters keep the orders of magnitude. The empirical literature usually reports different results of the theoretically expected. Comparing with previous findings, in in Matos *et al*. (2016b), only the volatility seems to be relevant, while Matos and Reis (2016) report the significance and the expected sign for both volatility and spread.

With respect to the estimation of the forward-looking part of the model, most of the sign of parameters associated with expectations support the economic intuition. First, the results show us how relevant is analyzing the monetary authority behavior considering its time-varying essence, because while in the second period only expected changes in current account significantly impacts reserves, in the following period we can evidence a significant influence of expectation of five macroeconomic variables. Table 4 summarizes the results reported in Table 3, to allow a better view on what direction each macroeconomic variable affects the reserves in each period.

Net public sector debt to GPD ratio is the macroeconomic variable whose change in expectations seems to be more relevant, to be significant in all periods, except for the second period. The results suggest that expected increases in government spending tend intuitively to affect the reserves in a more conservative direction by the Central Bank of Brazil. Only in the fourth period, this influence is negative.

Expected increases in the growth rate of industrial production and GDP affect during the period from April 2007 to the end of 2015, but in different directions. Expected increases in industrial production tend to stimulate the Central Bank of Brazil to increase reserves, while bad news associated with the Brazilian GDP growth suggest that the monetary authority should increase the amount of reserves.

Another variable which impacts the reserves over more than one period is SELIC rate, whose expectations increase are able to negatively influence the reserves between August 2003 and April 2005, but positively affect the level of reserves during the period associated with recent American mortgage crisis. Raising reserves also depend on expected increases in IGP–DI inflation index and R$/US$ exchange rate during the fourth period. Observing the variables associated with transactions with foreigners, in the second period the current account balance positively affects the reserves, while in the following period, reserves are adversely affected by increases in trade balance.

Table 4 - Summary of significant results (sign) of forward-looking buffer stock model estimation [a, b, c]

Estimations	Periods: subsamples based on multiple break tests				
	1st period (97 observations) From August 09, 2003 to April 11, 2005	2nd period (102 observations) From April 18, 2005 to March 26, 2007	3rd period (116 observations) From April 02, 2007 to June 15, 2009	4th period (98 observations) From June 22, 2009 to May 02, 2011	5th period (243 observations) From May 09, 2011 to December 28, 2015
$\Delta ipca^{exp}$					
$\Delta igpdi^{exp}$				(+)	
$\Delta igpm^{exp}$					
Δipc^{exp}					
$\Delta forex^{exp}$				(+)	
$\Delta selic^{exp}$	(-)		(+)		
$\Delta debt^{exp}$	(+)		(+)	(-)	(+)
Δgdp^{exp}			(-)		(-)
Δind^{exp}			(+)	(+)	
Δcur^{exp}		(+)			
$\Delta trade^{exp}$			(-)		
Δfdi^{exp}					
Δadm^{exp}					

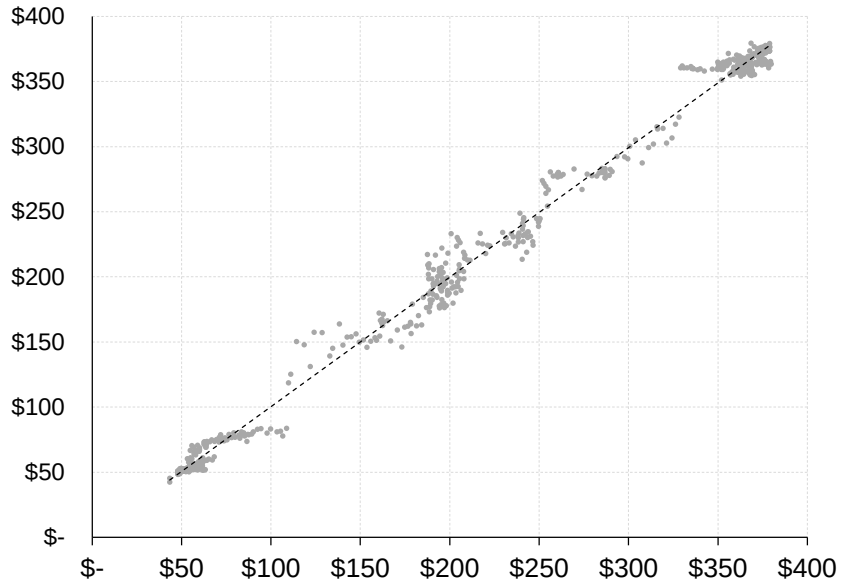
[a] Estimation results based on estimation over the period from June 09, 2003 to December 28, 2015. [b] GARCH models estimated through ARCH with normal distribution errors, using the Bollerslev-Wooldridge robust covariance coefficient heteroscedasticity of the residuals. [c] We specify the Newey and West (1987) covariance coefficients method, in order to get robustness to heteroskedasticity in mean equation estimations based on OLS method.

Our final analysis of the results reported in Table 3 is based on the complementary results. For all five samples of time, the Wald test, whose null hypothesis is the joint significance of the parameters associated only to the expectations, suggests that it is necessary to use the selected set of expectations in each period.

The explanatory power of the forward-looking model is greater than 0.6, with exception of the first and last periods. In all cases, there is a gain of explanatory power in relation to the benchmark buffer stock model, especially in the period of greatest turbulence of the time sample, which coincides with the American crisis that began in 2007. Comparing this metric with previous evidences, in Matos *et al*. (2016b), the adjusted R^2 is lower than 0.04, even considering the cross- effects of BRIC and in Matos and Reis (2016), when they take into account for cross-effects of other South American economies, this measure reaches out 0.23.

Another way to measure the fitting of our forward-looking model is to monitor in sample forecast errors. Figure 4 shows the dispersion of weekly total reserves by plotting the realized value on the horizontal axis and the theoretically predicted value on the vertical axis.

Figure 4 - Dispersion of weekly total reserves (US$ billion) for Brazil [a]



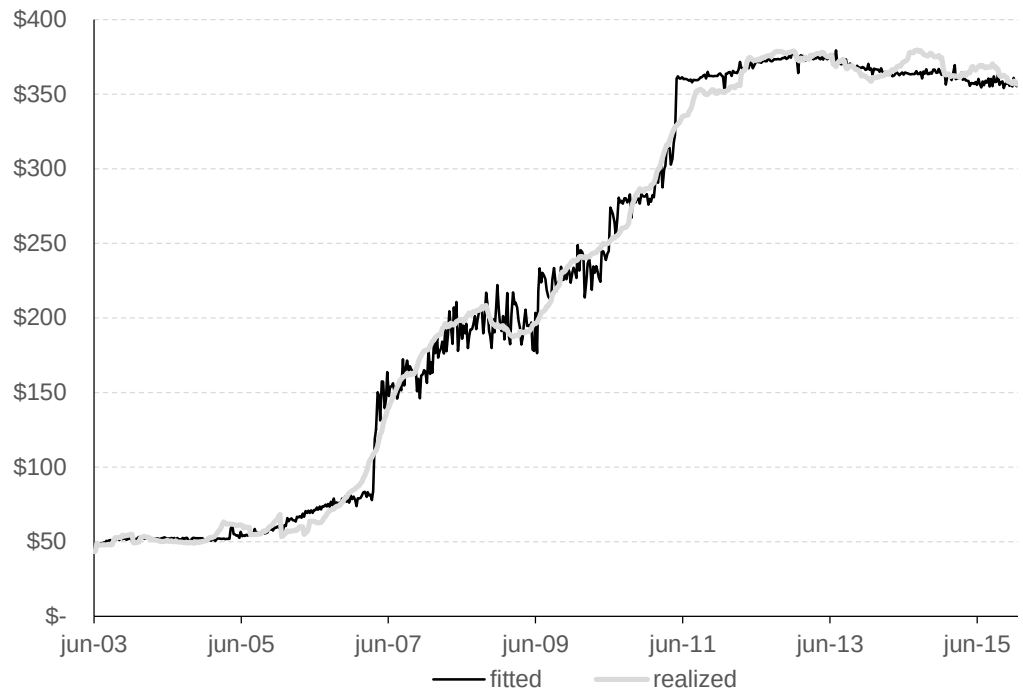
[a] This figure plots the weekly realized reserves (horizontal axis) versus its prediction based on forward-looking buffer stock model (vertical axis), during the period from June 09, 2003 to December 28, 2015.

There is clearly a good fitting of our model over time. Looking at the sub-periods during the period from early 2005 until mid-2007, the forecast errors assume greater absolute values, exceeding 10%, while during the last period, from May 2011 to December 2015, we have an adjustment even more appropriate of our model, with forecast errors ranging between -2% and 2%. The mean square error over the whole time was 6.5%.

Figure 5 shows the time trajectory of the weekly realized and predicted reserves in Brazil. An interesting application of this figure is to support an increasing policy (or decreasing) of reserves based on persistence able to characterize lack of protection (or excessive conservatism).

In this sense, the longer periods characterized by a sequence without interruptions of total reserves in a level below the theoretical optimum level are from 26 December 2005 to October 9, 2006 (42 weeks) and more recently, from 09 May 2011 to 26 December 2011 (34 weeks). In both periods, the average deficiency was approximately US$ 7 billion and US$ 15.6 billion respectively. In relation to periods characterized by longer and persistent excess conservatism, we highlight the periods between April 7, 2014 and December 22, 2014 (38 weeks) and between March 30, 2015 and November 3, 2015 (32 weeks). In both cases, excess conservatism assumed average values of US$ 10.1 billion and US $ 8.2 billion respectively.

Figure 5 - Weekly realized and fitted reserves (US$ billion) for Brazil [a]



[a] This figure plots the series of weekly realized reserves and its prediction based on forward-looking buffer stock model, during the period from June 09, 2003 to December 28, 2015.

4.5. Discussion

Because of the good performance of fitting, the assumptions of our microfundamented model and the results based on the individual and joint significance, we believe that our framework is useful to support Brazilian policy makers decisions about driving the stock of international reserves. First, we can infer based on trajectories reported in Figure 5 and from forecasting errors that the behavior of the monetary authority in practice has been close to provided by the theoretical framework. Second, based on the premise that the Central Bank of Brazil behaves as forward-looking when it decides on reserves level, we can show during the period between June 2003 and December 2015 that in a few specific time periods it is possible to evidence periods with more than six months without interruption characterizing an excess conservative sequence or a reserves deficiency sequence.

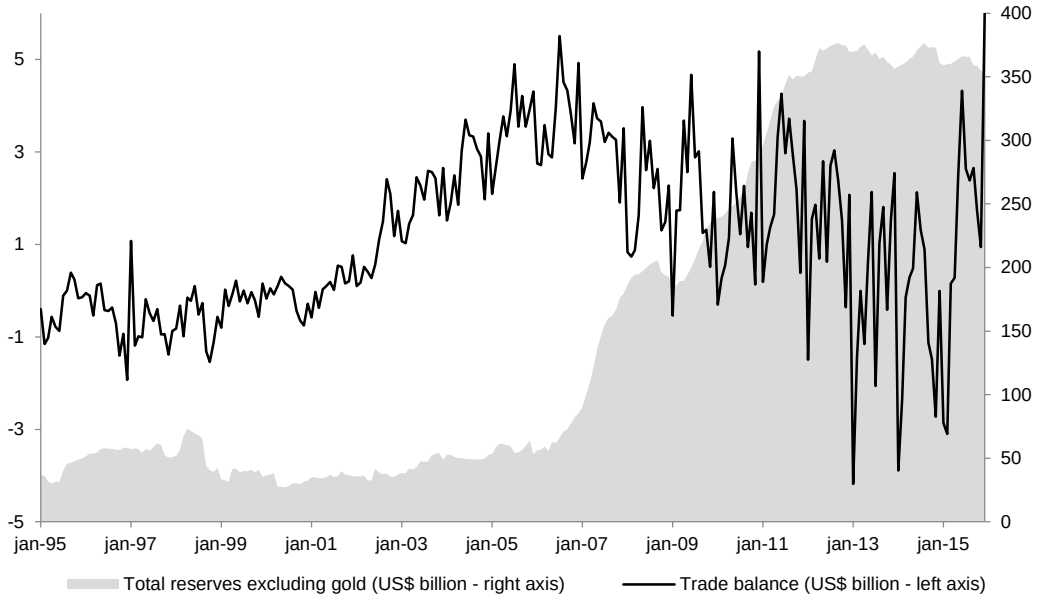
The relevance of this latest evidence should support related decisions in the current Brazilian economic scenario, in which there is a crisis with ethical–moral, political–economic and legal–institutional fundamentals. According to Brazilian press, there are recent signs of the Brazilian federal government aiming to make use of this indispensable level of reserves for income-generating purposes useful to dealing with the deficits or debts. We note that, despite the recent persistence of excess conservatism, observed over 32 weeks between March 30, 2015 and November 3, 2015, the average amount of the excess is US$ 8.2 billion, only.

Therefore, besides being an unconstitutional measure, excess of the average values are much smaller than the values of the primary deficit in 2015, R$ 111 billion (1.88% of GDP) and negligible if compared to the nominal deficit, R$ 613 billion (10.34% of GDP). In terms of public sector debt, the net and gross correspond to approximately 36% and 67% of GDP respectively.

Another argument that supports our suggestion to keep this policy of the Central Bank of Brazil is the path of reserves along with the trajectory of current indicators involving transactions with external agents. Figures 6, 7 and 8 show the trajectory of the reserve level in US$ billion, comparing it with the trajectory of the trade balance, the current account and gross external debt of the federal government, respectively.

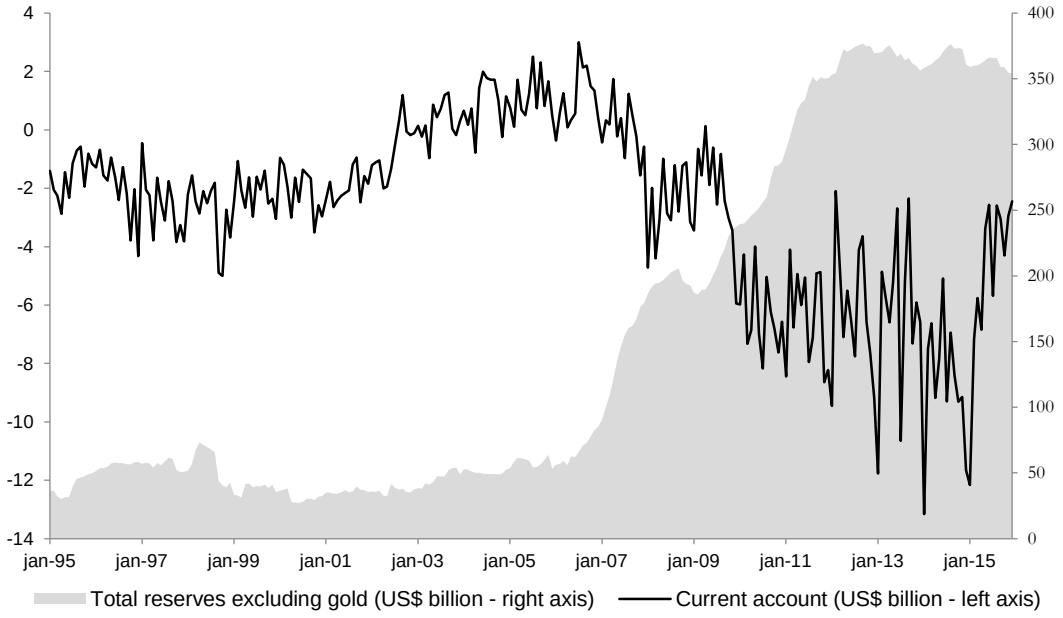
According to Figure 6, more specifically, from January 2012, the standard deviation is US$ 2.13 billion, higher than the standard deviation of the period ending in December 2011, about US$ 1.7 billion. On average, the trade balance fell more than half from US$ 1.28 billion a month during 1995-2011, to US $ 0.60 billion, from 2012. Comparing both periods, the average reserves are US$ 100 billion and US$ 366 billion respectively.

Figure 6 - Monthly total reserves excluding gold and monthly trade balance for Brazil from January 1995 to December 2015. Data sources: Central Bank of Brazil and FRED



As well as the worsening of the trade balance in recent years suggests greater protection via reserves, when we observe Figure 7, again there appears to be justification for an increase in recent years of the reserve level.

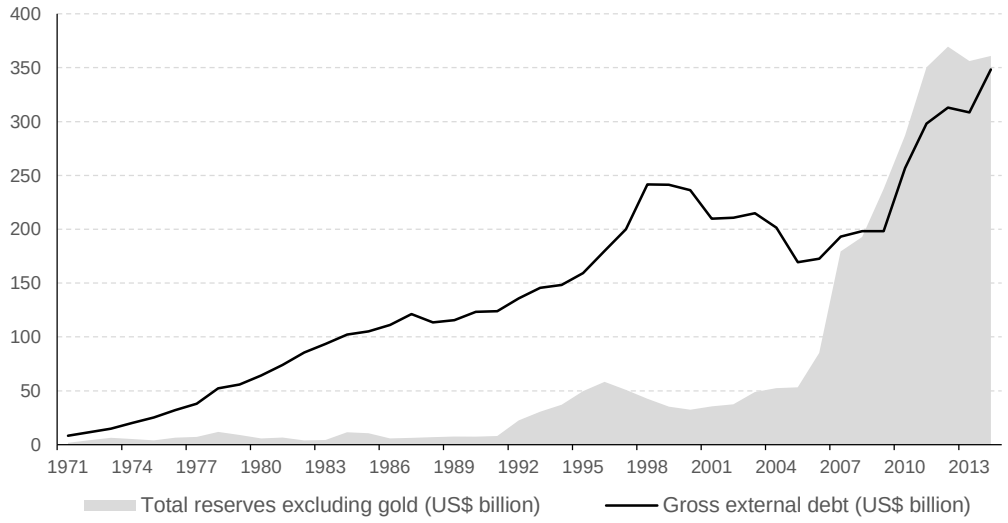
Figure 7 - Monthly total reserves excluding gold and monthly current account for Brazil from January 1995 to December 2015. Data sources: Central Bank of Brazil and FRED



The balance of current account takes positive values in most months during the years 2003 to 2007. Thereafter it reports successive and high negative values. From May 2009, we can see a deficit every month, reaching out the maximum value of US$ 13 billion of deficit in January 2014. When one separates the period from January 1995 to December 2015, in two subperiods, using May 2009 as the threshold, the average reserves are $ 68 billion and $ 330 billion respectively.

Finally, Figure 8 shows us how annual trajectories of reserves and the gross external debt of the country behaved for an even longer period of time, 1971-2014. Considering the period from 2007, since the reserves have increased more than 100% from 2006 to 2007, we have a geometric mean increase of 10.5 % per year, compared to an average geometric growth of 8.8% of gross foreign debt. This behavior of reserves seems to be a need to monitor the growth of the gross external debt, to compensate decades in which debt was quite larger than reserves. In some years during the 80s, foreign debt reached the value of 20 times the amount of total reserves of the country.

Figure 8 - Annually total reserves excluding gold and annually gross external debt for Brazil from 1971 to 2014. Data sources: Central Bank of Brazil and FRED



5 CONCLUSION

The stock of international reserves of a country must be seen as a public asset useful to ensure continuity of economic activity and preserve financial stability, mainly for emerging economies.

In Brazil, one can evidence interesting numbers about this protection. Following the role of thumb about the ratio total reserves to imports, Brazil has one of the highest values: reserves are able to cover more than nine months' worth of imports, instead of three months. Considering only the last decade, the level of total reserves ranges from US$ 53.8 billion in January 02, 2006 to more than US$ 356 billion, a raise of more than 560%. For the other side, the ratio reserves to GDP of 15% for Brazil in December 2015 is close to the ratios of Chile, India and South Africa. In sum, it is not possible infer based on those numbers about how appropriate or not is the level of Brazilian reserves, unless one can extract an optimal theoretical path that allows to characterize excess or lack of reserves.

Because of the performance of fitting, the assumptions of our model and the results based on the individual and joint significance, we believe that our microfundamented time varying forward-looking framework is useful to support Brazilian policy makers decisions about driving the stock of international reserves. We may highlight the role played by net public sector debt to GPD whose change in expectations seems to be more relevant, due to its significance in almost all periods.

Our contribution is especially relevant for Brazil, because the recent context characterized by severe local crisis and a nominal deficit in December 2015 of R$ 613 billion, more than 10% of GDP and given the lack of prospects for improvement of the main economic

fundamentals of the country, such as inflation, GDP, employment and investment. In this scenario, we claim that Brazilian society needs to be protected at least by the current precautionary level of international reserves, an achievement of Brazilian people through effort of Brazilian monetary authority.

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We add to the discussion about a puzzle in international finance: the excess reserves in emerging countries, by proposing a methodological innovation aiming to model the optimal time path of the amount of international reserves in Brazil. Based on previous evidences for this economy, we extend the microfundamented benchmark buffer stock model proposed by Frenkel and Jovanovic (1981), assuming a time-varying forward-looking behavior of Brazilian monetary authority. We perform an empirical exercise, by applying this model for a weekly database from June 02, 2003 to December 28, 2015, taking into account multiple break dates as thresholds useful to identify five subperiods. According to our main findings, net public sector debt to GPD ratio, SELIC rate and GDP growth are some of the macroeconomic variable whose change in expectations seems to be more relevant to influence level of reserves. Our forward-looking extension provides a gain of explanatory power in relation to the benchmark model, mainly in the period of greatest turbulence from April 2007 to June 2009. We believe that our framework can be useful to support Brazilian policy makers decisions about driving the stock of international reserves.

Av. da Universidade, 2700 • Benfica • Fone/Fax:(085) 3366.7751
Cep: 60.020-181 • Fortaleza - CE - Brasil
http://www.caen.ufc.br • e-mail: caen@caen.ufc.br