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The Antitrust Mixed Logit Model

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This paper presents the Antitrust Mixed Logit Model (AMLM), a novel methodology that shows how to calibrate the parameters of a mixed-logit demand model and simulate the competitive effect of horizontal mergers. The major advantage over the simpler Logit version (the Antitrust Logit Model, ALM, developed by Werden and Froeb,1994) is flexibility, resulting in more plausible elasticities and consequently more precise predictions about merger effects. Moreover, unlike the econometric approaches, the AMLM shares with the ALM the attributes that are particularly appealing to antitrust agencies, given time and data constraints they usually face: low data requirement and high computational speed. This model is applied to simulate mergers in the U.S. ready-to-eat cereal industry.

Keywords- Antitrust, Mixed Logit, Merger Simulation, Competition Analysis

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I-INTRODUCTION

The Antitrust Logit Model (ALM) developed by Werden and Froeb (1994) has been widely applied to predict the competitive effects of horizontal mergers in product-differentiated industries. The ALM does not place great demands on the data set and is fast to compute and– we only need information on prices, market shares and two exogenously given parameters (usually price elasticities). These attributes make this calibration methodology particularly appealing to antitrust agencies and merging firms, given time and data constraints they usually face. However, it is well known that the Logit demand model places very restrictive limitations on own and cross price elasticities, which constitute critical economic parameters in the evaluation of merger effects.

This paper presents the Antitrust Mixed Logit Model (AMLM), a new methodology whose main contribution is to show how to calibrate the parameters of a mixed-logit demand model. After the calibration step the model follows most merger simulation models found in the industrial organization literature, i.e. it assumes Bertrand competition in order to obtain marginal costs and simulate mergers. The major advantage over the logit version is the flexibility of the mixed logit demand, which generates more realistic patterns of substitution between goods and consequently more precise predictions about merger effects. Moreover, unlike the econometric approaches to uncover the parameters of a mixed-logit demand (e.g. Berry, Levinsohn and Pakes, 1995), the AMLM shares with the ALM the attributes that are particularly appealing in merger investigations: low data requirement and high computational speed. Indeed, the data requirement to implement the AMLM is almost the same as the ALM and the computational burden is low.

This paper is organized as follows. Section II presents the demand model. The following section introduces the supply side, in which firms compete à la Bertrand. In section IV the AMLM and the ALM are compared. An application of the ALML is presented in section V. Finally, additional comments can be found in the last section.

II – Mixed Logit Demand

In this section, I shall describe a discrete-choice demand model with one random coefficient and propose a new methodology to uncover its parameters[1]. Consumers rank products according to their characteristics and prices. There are *N+1* choices in the market, *N* inside goods and one reference good (or outside good). Consumer *i* chooses brand *j*, given price p_j, unobserved characteristics and quality (summarized by the scalar δ_j), and unobserved idiosyncratic preferences ε_{ij}, according to the following utility function:

(1) $$u_{ij} = g(\alpha, v_i) p_j + \delta_j + \varepsilon_{ij}$$

where $g(\alpha, v_i)$ is a random coefficient that represents consumer *i*'s marginal utility (or disutility) of price, which is a function of the parameter α and a consumer-specific term v_i. Moreover, δ_j can be interpreted as the mean utility of product *j* derived from product attributes other than prices. The utility derived from the consumption of the outside good can be normalized to zero u_{i0}=0. Assuming

[1] It will be made clear why the restriction on the number of random coefficients is necessary in the methodology developed in this paper. The limitations arising from using a mixed logit model with only one random coefficient rather than its more general version with more than one random coefficient deserves further attention. However, it is important to stress that this restricted mixed logit model is superior to logit and nested logit models, which impose severe restrictions on price elasticities (see Nevo, 2000a). Song (2007) uses a mixed logit with one random coefficient as a basis of comparison with pure characteristics models.

that ε_{ij} has a Type I Extreme Value distribution, the probability of individual i choosing good j (σ_{ij}) takes the familiar logit form

$$\text{(2)} \qquad \sigma_{ij}(\alpha, p, \delta, v_i) = \frac{\exp(g(\alpha, v_i) p_j + \delta_j)}{1 + \sum_{m=1}^{N} \exp(g(\alpha, v_i) p_m + \delta_m)}$$

In addition, taking the expected value of this probability with respect to the distribution of the v_i's yields the probability of good j being chosen (σ_j), which is given by

$$\text{(3)} \qquad \sigma_j(\alpha, p, \delta) = E_v[\sigma_{ij}(\alpha, p, \delta, v_i)]$$

In turn, the choice probabilities conditioned on one of the inside goods being chosen (σ_{jI}), i.e. the inside good share of good j (s_{jI}) is

$$\text{(4)} \qquad s_{jI} = \sigma_{jI}(\alpha, p, \delta) = \frac{\sigma_j(\alpha, p, \delta)}{\sigma_I(\alpha, p, \delta)}$$

where σ_I is the probability of one of the inside goods being chosen. Thus the model implies that the inside good share of good j depends on the parameter α, and N-dimensional column vectors p and δ, that collect all p_j's and δ_j's respectively.

Empirical Strategy to Uncover Demand

I assume that the researcher has information on (inside) market shares, prices, the distribution of the consumer-specific term v_i and two price elasticities: aggregate elasticity and the elasticity of one inside good. For the demand model presented above the implied own-price elasticity for given good l is given by

(5) $$\eta_{ll}(\alpha,p,\delta)=\frac{p_j}{s_{ll}.\sigma_I}E_v[g(\alpha,v_i).\sigma_{il}(\alpha,p,\delta,v_i)(1-\sigma_{il}(\alpha,p,\delta,v_i)]$$

In turn, the aggregate demand of all inside goods η_I, also known as the aggregate elasticity, η_I is given by

(6) $$\eta_I(\alpha,p,\delta)=\frac{E_v[g(\alpha,v_i).P_i(\alpha,p,\delta,v_i).\sigma_{i0}(\alpha,p,\delta,v_i)]}{\sigma_I}$$

where $P_i=\sum_{m=1}^{N}(\sigma_{im}.p_m)$ and $\sigma_{i0}(\alpha,p,\delta,v_i)=\frac{1}{1+\sum_{m=1}^{N}\exp(g(\alpha,v_i)p_m+\delta_m)}$ is the probability consumer *i* choosing the outside product.

Note that the system of equations formed by Equations (4), (5) e (6) can be rewritten as

(7) $$s_{jI}=\frac{|\eta_I|.\sigma_j(\alpha,p,\delta)}{E_v[|g(\alpha,v_i)|.P_i(\alpha,p,\delta,v_i).\sigma_{i0}(\alpha,p,\delta,v_i)]};\ j=1,....N$$

(8) $$|\eta_{ll}|=\frac{|\eta_I|p_l}{s_{lI}}\frac{E_v[|g(\alpha,v_i)|.\sigma_{il}(\alpha,p,\delta,v_i)(1-\sigma_{il}(\alpha,p,\delta,v_i)]}{E_v[|g(\alpha,v_i)|.P_i(\alpha,p,\delta,v_i).\sigma_{i0}(\alpha,p,\delta,v_i)]}$$

This system is key to the empirical strategy proposed in this paper. Indeed, notice that as assumed the researcher observes (inside) market shares s_{jI}, prices, the distribution of the consumer-specific term v_i, the aggregate elasticity η_I and the elasticity of one good η_{ll}. Therefore, since the system is formed by *N+1* equations we can uncover the *N+1* unknowns (*N*-dimensional vector δ plus the scalar α)[2]. Based on Berry, Levinsohn and Pakes (1995), BLP henceforth, I propose an algorithm to solve this system of equations (see the appendix).

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[2] If α is vector of dimension greater than one, and not a scalar as assumed here, or if we had more than one random coefficient, the system would certainly be under identified. For this reason we have to posit a mixed logit model with only one random coefficient with only one parameter. Whether this is a plausible model is largely an empirical question. Notice also that α is deterministic and therefore it does not have a standard error. The model could be easily extended to accommodate more random coefficients however the another elasticity given a priori would have to be brought to the empirical strategy.

III – Supply and Merger Simulation

Uncovering demand parameters is not enough to perform merger simulation, we also have to specify how firms compete. I follow the commonly adopted assumption that firms choose prices simultaneously in a one-shot game, i.e. the market outcome is the result of a Bertrand game.

Supply

First, assume that each firm *f* produces a subset F_f of the goods sold in this market. If firms behave according to Bertrand, it can be shown that the price of product *j* produced by firm *f* at (constant) marginal cost c_j must satisfy the following equation

(9) $$\sigma_j + \sum_{r \in F_f} (p_r - c_r) \frac{\partial \sigma_r}{\partial p_j} = 0 \; ; j=1,2,\ldots,N$$

Or, equivalently,

(10) $$\sigma - (\Omega \Delta [p - c]) = 0$$

where σ, *p* and *c* are Nx1 vectors collecting σ_j's, prices e marginal costs respectively. In addition, Δ and Ω are *N*x*N* matrix whose typical element (*j,r*) is defined as follows

$$\Delta_{jr} = -\frac{\partial \sigma_r}{\partial p_j} \text{ and}$$

$$\Omega_{jr} \begin{cases} 1 & \text{if } r \text{ e } j \text{ are produced by the same firm} \\ 0 & \text{Otherwise.} \end{cases}$$

The outside good pricing decision is assumed to be exogenous and therefore does not interact strategically with the pricing decision of the inside goods. Note that (10) is flexible enough to accommodate different market structures. The first structure is the single firm product, in which the firm can only control the price of its unique brand. The second is the multi-product firm, in which the firm internalizes the price decision of different brands. A third example is a monopoly, where one firm produces all the varieties offered in the market.

Simulating Mergers

One implicit assumption in merger simulation is that observed pre-merger (and also post-merger) prices are generated by the outcome of Bertrand competition between firms. Therefore, Equation (10), evaluated at pre-merger prices, is given by

(11) $\sigma(p^{pre}) - (\Omega^{pre}\Delta(p^{pre})[p^{pre} - c]) = 0$.

Hence, marginal costs can be uncovered from the following equality

(12) $c = p^{pre} - [(\Omega^{pre}\Delta(p^{pre}))^{-1}\sigma(p^{pre})]$

Notice that p^{pre} represents the observed pre-merger prices and that Ω^{pre} is constructed using pre-merger ownership structure. Once we have demand and supply parameters (α, δ and c) it is possible to calculate the equilibrium prices resulting from the new ownership structure arising from the merger. Indeed, the predicted post merger prices (p^{post}) is the solution of the following system of equations

(13) $\sigma(p^{post}) - (\Omega^{post}\Delta(p^{post})[p^{post} - c]) = 0$

where Ω^{post} is constructed using the post merger ownership structure.

IV – The Simple Logit (The Antitrust Logit Model)

In this subsection I present the simplest discrete-choice model: the Logit. This exposition serves the purpose of a model that yields analytical formulas and consequently is simpler. However, as well documented in the discrete-choice literature (see BLP), the Logit demand model places very restrictive limitations on own and cross price elasticities, which constitute critical parameters in the economic evaluation of innovation, mergers and entry of new products.

In the Logit case, we can assume without loss of generality that $g(\alpha, v_i) = -\alpha$. Then

(14) $$\sigma_j(\alpha, p, \delta) = \frac{\exp(-\alpha p_j + \delta_j)}{1 + \sum_{m=1}^{N} \exp(-\alpha p_m + \delta_m)}$$

In turn, the choice probabilities conditioned on one of the inside goods being chosen (σ_{jI}), i.e. the inside good share of good j (s_{jI}) is

(15) $$s_{jI} = \sigma_{jI}(\alpha, p, \delta) = \frac{\sigma_j(\alpha, p, \delta)}{\sigma_I(\alpha, p, \delta)}$$

The Logit also implies an analytical formula for the aggregate and own-price elasticities. Indeed,

(16) $\eta_{II}(\alpha, p, \delta) = -\alpha p_j [1 - \sigma_j]$, and

(17) $\eta_I(\alpha, p, \delta) = -\alpha \bar{p} \sigma_0$

where, $\bar{p} = \sum_{m=1}^{N} \sigma_m p_m$ is a weighted average price. The system of equations – Equations (15), (16) e (17) simplifies to the following system of equations[3]:

(18) $$\ln[s_{jI}(1-\sigma_0)] - \ln[\sigma_0] = -\alpha p_j + \delta_j \qquad ; \quad j=1,\ldots N$$

(19) $$|\eta_j| = \frac{[\alpha \bar{p}(1-s_{jI}) + |\eta_I| s_{jI}] p_j}{\bar{p}}$$

where $\sigma_0 = \frac{|\eta_I|}{\alpha_{cal}\,\bar{p}}$. This system is much simpler than its version for the more general model imposed in the last section. Given prices *p* and two elasticities η_I and η_j we can directly solve for α from Equation (19), giving $\alpha = \frac{|\eta_j|\bar{p} - |\eta_I| s_{jI} p_j}{p_j \bar{p}(1-s_{jI})}$. Once α is determined, we can find the corresponding δ_j's from (18), which are given by $\delta_j = \ln[s_{jI}(1-\sigma_0)] - \ln[\sigma_0] + \alpha p_j$. Note that the simple logit (developed by Werden and Froeb, 1994, and known as the antitrust logit model) is a particular version of the mixed logit model developed in this paper. It is important to stress that this more general model is very useful for demand analysis, especially merger simulation, as it accommodates more reasonable patterns of substitution between products.

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[3] The system is linear in the unknowns (δ, α)

V – AN EXAMPLE

In order to illustrate the methodology, I use data on the ready-to-eat cereal industry. However, it should be noticed that the objective of this section is to illustrate the methodology proposed in this paper rather than providing a detailed study of the ready-to-eat cereal industry. Nonetheless, an application of this methodology that takes into consideration all or most of the idiosyncrasies of this industry would be an interesting extension of this work.

The data set is a cross-section of fifty top selling brands in the U.S in 1992. The summary statistics are presented below[4]. The data set reports information on (inside) shares and prices. To construct the shares it is assumed that the set of inside goods is composed of all the top fifty best selling brands. Thus, this implies that the outside good is representative of all other brands and other substitutes not included in the top fifty best selling list.

Table I

Summary statistics for Ready-To-Eat Cereal Industry in the U.S – 1992

	Mean	*Std Dev*	*Variance*	*Min*	*Max*
Share	0.0152	0.0102	0.0001	0.0067	0.0567
Price ($/lb)	2.9830	0.4916	0.2416	1.7700	3.9600

Source: Descriptive statistics for variables available in the data set mentioned above.

I follow Berry, Levinsohn, and Pakes (1999) and parameterize the consumer marginal utility for price according to the functional form given

[4] This data set was constructed by Matt Shum, who gently allowed me to use it.

by $g(\alpha, v_i) = -\frac{\alpha}{v_i}$, where the consumer-specific term v_i represents household income, whose distribution is obtained from the 1992 Current Population Survey (CPS). In order to simplify the computation of the mixed logit model, I made a few simplifications regarding this distribution. I have divided the income space into intervals of the same size (2500 USD) and computed the frequencies of each interval. Then, I discretize the distribution assuming that the average income in each interval is representative of all individuals included in this interval. In the end, we have 21 income levels and thus 21 consumer types. The discretization avoids the need for numerical integration (e.g. quadrature methods) or simulation methods (as employed by BLP) to compute the markets shares in Equation (3). This is done to reduce the computational burden. Notice that if the researcher is not willing to make these simplifications, the methodology model outlined in section III can certainly accommodate different distributional assumptions for income such that quadrature or simulation methods can be used.

In the first stage of the, I posit that $\eta_I = -0.35$ and the elasticity[5] of one inside good[6] (KG Corn Flakes) $\eta_{II} = -3$. Then we are able to uncover *N+1*-dimensional vector (δ, α). I find that α is 41979.102 , from which we can derive the distribution of the price coefficients (in absolute values) across consumers. This distribution is given by the distribution of the ratio $-\frac{\alpha}{v_i}$. We can also construct descriptive statistics for the δ_j's. These results are summarized in Table II below.

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[5] These numbers are similar to those found in Nevo (2001)

[6] These values compose the prior information set. I could have used other values for the aggregate and own-price elasticities to perform robustness checks. This is left for future developments of this work.

Table II

Summary statistics of stage 1 results

	Mean	Median	Max	Min
Price coefficient (in modulus)	2.001	0.799	16.791	0.399
Mean utilities (δ_j's)	4.157	4.168	5.614	1.758

The distribution of the price coefficient has mean 1.982 and median 0.791, implying that the distribution is not symmetric around its mean. The average (across brands) mean utility is 4.157. The distribution is approximately symmetric around the mean since the mean and the median are approximately equal.

Merger Simulation

An advantage of structural estimation is that, once the parameters of interest are determined, one can simulate the effect of different market environments using the structural model. The merger simulation goes as follows. Determine the demand parameters using the empirical strategy developed in this paper. Next, use the observed equilibrium prices before the merger to uncover marginal costs from Equation (12) and next find the equilibrium prices resulting from the new ownership structure using equation (13).

Tables IV shows the results from all possible merger between two firms. The first column indicates the firms involved in the simulated merger. The other columns present descriptive statistics of the prices changes resulting from the simulation. For instance, the model predicts that a merger between General Mills and Kelloggs would increase (share-weighted) industry average price from 2.887 to 3.315, which represents a 12.9% increase[7]. At the brand-level, the variety that presents the highest price variation (GM Triples) belongs to one of the merging firms, as expected, and exhibits a significant price increase (39.35%). The merger between *Kelloggs* and *Nabisco* would imply a moderate increase in the industry average price (1.85%). However, internalizing competition allows the new merged firm to charge a price 27% higher for the *Big Biscuit Shd* brand, which belongs originally to Nabisco. In turn, we should not expect significant anti-competitive effects from the Nabisco-Post Merger, since the industry average price increase is very small (0.53%) and no brand has its price inflated by more than 3.53%.

Table IV

Simulation Results

Merging Firms	Post-Merger Average Prices	Pre-Merger Average Prices	Increase in avg. Prices (%)	Largest price variation across brands(%)	Brand Name
KG/GM	3.315	2.887	12.909	39.348	*GM Triples*
KG/NB	2.942	2.887	1.852	27.030	*NB Big Biscuit Shd*
KG/PT	2.997	2.887	3.653	33.305	*PT Grape Nuts*
KG/QK	2.958	2.887	2.389	36.075	*QK Popeye*
KG/RL	2.912	2.887	0.841	22.432	*RL Muesli*
GM/NB	2.938	2.887	1.725	21.926	*NB Big Biscuit Shd*
GM/PT	2.985	2.887	3.276	23.460	*PT Grape Nuts*
GM/QK	2.948	2.887	2.068	23.038	*QK 100% Natural*
GM/RL	2.911	2.887	0.826	19.453	*RL Muesli*
NB/PT	2.903	2.887	0.530	3.529	*NB Big Biscuit Shd*
NB/QK	2.897	2.887	0.345	2.163	*QK Popeye*
NB/RL	2.892	2.887	0.148	1.746	*RL Muesli*
PT/QK	2.906	2.887	0.657	4.355	*QK Popeye*
PT/RL	2.895	2.887	0.252	3.017	*RL Muesli*
RL/QK	2.892	2.887	0.157	1.606	*RL Muesli*

V. FINAL REMARKS

The Antitrust Logit Model (ALM) developed by Werden and Froeb (1994) has been widely applied to predict the competitive effects of horizontal mergers in product-differentiated industries. The ALM does not place great demands on the data set and is fast to compute and– we only need information on prices, market shares and two exogenously given parameters (usually price elasticities). These attributes make this calibration methodology particularly appealing to antitrust agencies and merging firms, given time and data constraints they usually face. However, it is well known that the Logit demand model places very restrictive limitations on own and cross price elasticities, which constitute critical economic parameters in the evaluation of merger effects.

This paper presents the Antitrust Mixed Logit Model (AMLM), a new methodology whose main contribution is to show how to calibrate the parameter of a mixed-logit demand. After the calibration step the model follows most merger simulation models found in the industrial organization literature, i.e. it assumes Bertrand competition in order to obtain marginal costs and simulate mergers. The major advantage over the logit version is the flexibility of the mixed logit demand, which generates more realistic patterns of substitution between goods and consequently more precise predictions about merger effects. Moreover, unlike the econometric approaches to uncover the parameters of a mixed-logit demand (e.g. Berry, Levinsohn and Pakes, 1995), the AMLM shares with the ALM the attributes that are particularly appealing in merger investigations: low data requirement and high computational speed.

Indeed, the data requirement to implement the AMLM is almost the same as the ALM and the computational burden is low.

APPENDIX

One possible method to find the solution of the system is to employ commonly applied algorithms that search for the solution directly in the (δ,α) space. However, this would be computationally inefficient. Recall that one of the main motivations of the discrete-choice model relies on its the ability to deal with markets characterized by the presence of many brands. If we had 100 brands, for example, the algorithm would be searching directly in a space with dimension 101.

In order to deal with this dimensionality problem, we can take advantage of an important result derived in BLP. Given the parameter α and p the mapping defined pointwise by

$$T(s,\alpha,p)[\delta_j] = \delta_j + \ln(\bar{s}_{jI}) - \ln(s_{jI}(\alpha,p,\delta))$$

is a contraction mapping with modulus less than one. Therefore, we can improve computational efficiency by concentrating the search. Shortly, the algorithm goes as follows. The first step initiates the outer loop, which begins with a value of α', solve for the implied $\delta'(\alpha')$ by applying the contraction mapping algorithm (inner loop) to the sub-system formed by the N equations in (7). Then we calculate the implied elasticity of one of the inside goods $\eta_{ll}(\alpha',p,\delta')$ and then check whether equation (8) is satisfied. In this last step we verify how large is the distance between the prior information on the elasticity η_{ll} and the implied $\eta_{ll}(\alpha',p,\delta')$. If this α' does not imply a close enough distance, measured by $|\eta_{ll}| = \frac{|\eta_I| p_l}{s_{lI}} \frac{E_v[|g(\alpha, v_i)|.\sigma_{il}(\alpha, p, \delta, v_i)(1-\sigma_{il}(\alpha, p, \delta, v_i)]}{E_v[|g(\alpha, v_i)|.P_i(\alpha, p, \delta, v_i).\sigma_{i0}(\alpha, p, \delta, v_i)]}$, we repeat this process, by reinitiating the outer loop, until convergence has been attained[8].

[8] Thus, no matter how large is *N* (number of brands) the algorithm searches directly in a one-dimensional space.

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